Space Vector PWM

Anandarup Das

Asst. Professor

Room-402A, Department of Electrical Engineering, IIT Delhi.

anandarup@ee.iitd.ac.in

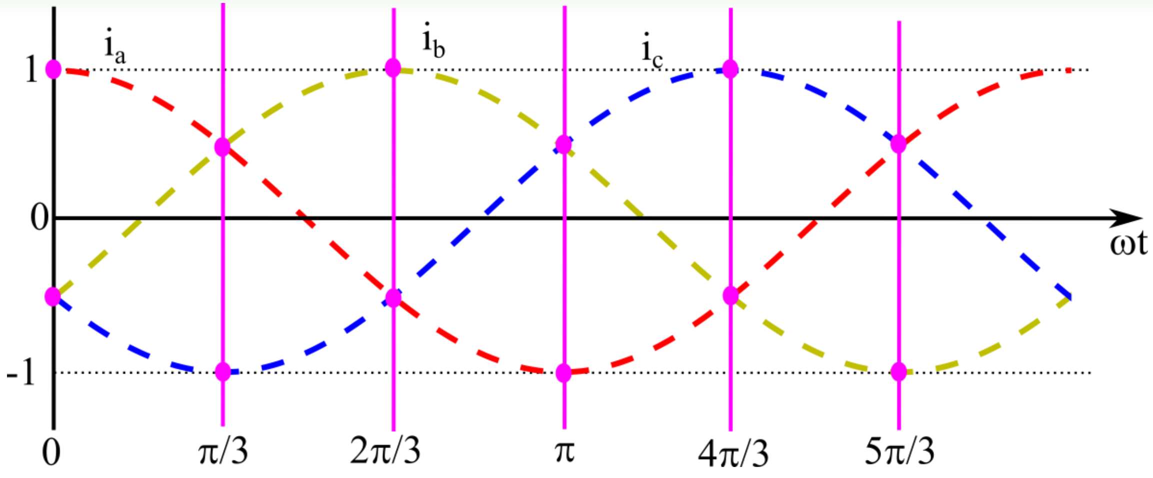
Space vectors

- The origin of space vectors lies in rotating mmf in machines.
- The resultant mmf for a three phase system is a rotating mmf having a fixed magnitude and direction at every instant of time.
- Space vector is a mathematical concept which is useful for visualizing the effect of three phase variables in space.

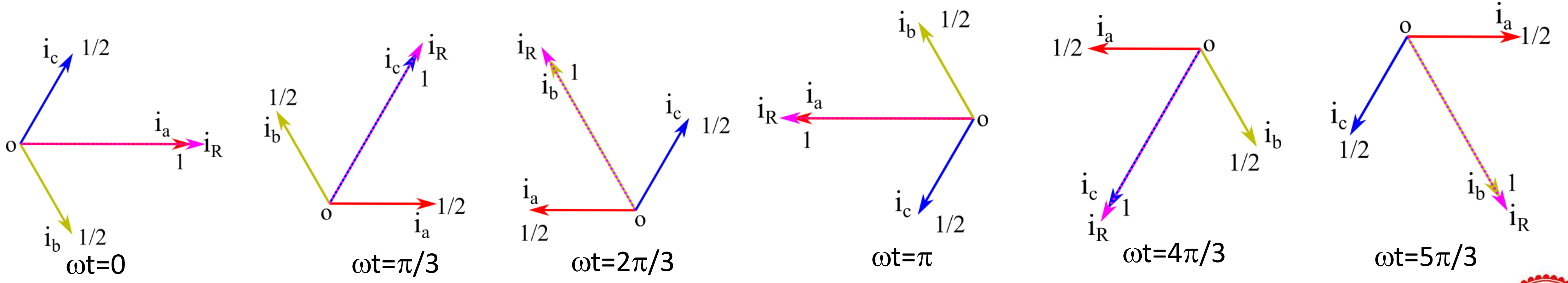
Space vectors

- Resultant space vector for load phase voltage or current are defined as,
  - $\boldsymbol{V_R(t)} = \frac{2}{3}\left[v_{An}(t) + v_{Bn}(t)e^{\frac{j2\pi}{3}} + v_{Cn}(t)e^{\frac{j4\pi}{3}}\right]$
  - $\boldsymbol{I_R}(\boldsymbol{t}) = \frac{2}{3}\left[i_A(t) + i_B(t)e^{\frac{j2\pi}{3}} + i_C(t)e^{\frac{j4\pi}{3}}\right]$
- The space vectors ***V_R(t)*** or ***I_R(t)*** have both magnitude and angle. Individual voltages/currents can be balanced or unbalanced and need not be sinusoidal.

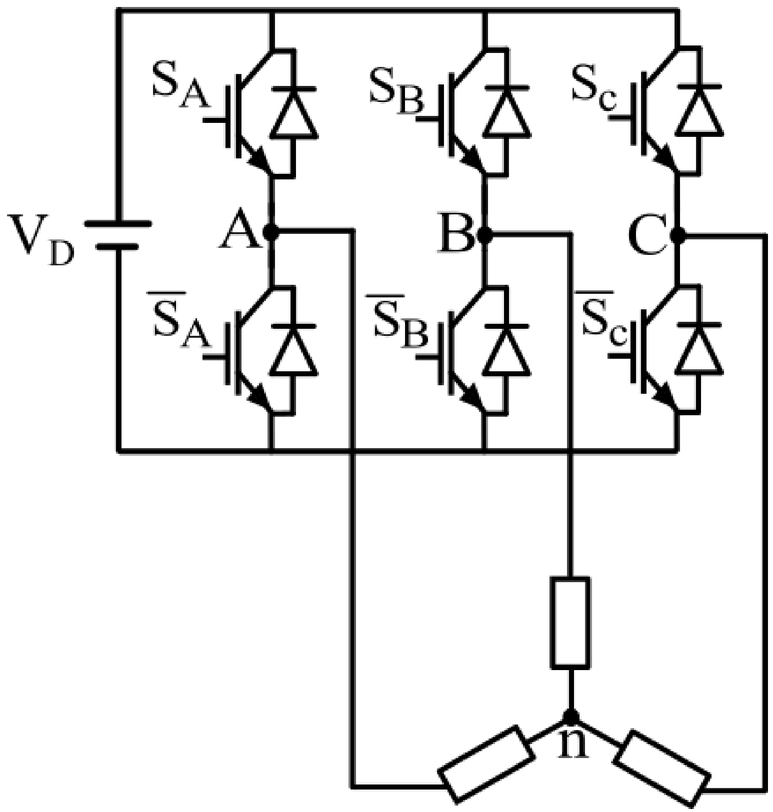
Current space Vector



- For the sinusoidal three phase currents, the resultant current space vector is shown.
- The resultant space vector (pink) is rotating at a uniform speed and having a constant radius.



Space vectors



- The pole voltage of one phase of the converter has two switching states: 1 (=V_D) and 0(=0).
- The converter has total eight switching states (2*2*2=8). These are: (000,111,100,110,010,011,001,101).
- There are six active vectors and two zero vectors.
- What is the load phase voltage space vector for 100 combination?

Space vector for 100 combination

- $v_{AO}(t) = VD \quad , v_{BO}\ (t) = 0, v_{CO}(t) = 0$
- $v_{An}(t) = \frac{2}{3}v_{AO}(t) - \frac{1}{3}v_{BO}(t) - \frac{1}{3}v_{CO}(t) = \frac{2}{3}V_D$
- $v_{Bn}(t) = \frac{2}{3}v_{BO}(t) - \frac{1}{3}v_{CO}(t) - \frac{1}{3}v_{AO}(t) = -\frac{1}{3}V_D$
- $v_{Cn}(t) = \frac{2}{3}v_{CO}(t) - \frac{1}{3}v_{AO}(t) - \frac{1}{3}v_{BO}(t) = -\frac{1}{3}V_D$

- $\boldsymbol{V_R(t)} = \frac{2}{3}\left[v_{An}(t) + v_{Bn}(t)e^{\frac{j2\pi}{3}} + v_{Cn}(t)e^{\frac{j4\pi}{3}}\right] = \frac{2}{3}V_D e^{j0}$

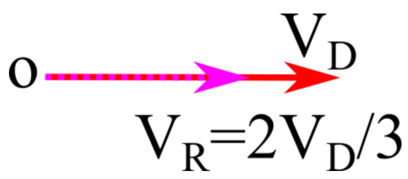
- Similarly we can deduce the resultant space vector for other combinations.

Space vector for all combinations

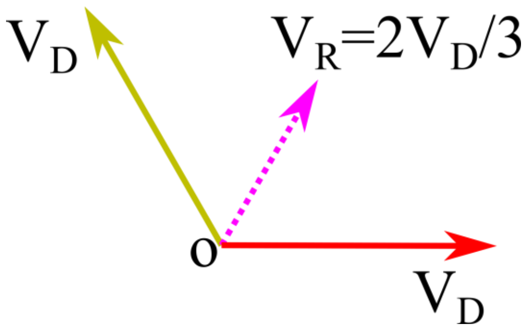
Space Vector	Switching States	Resultant space vector (***V_R(t)***)	
V0	000	$\vec{V_0} = 0$	Zero Vector
V1	100	$\vec{V_1} = \frac{2}{3} V_D e^{j0}$	Active Vector
V2	110	$\vec{V_2} = \frac{2}{3} V_D e^{j\pi/3}$	
V3	010	$\vec{V_3} = \frac{2}{3} V_D e^{j2\pi/3}$	
V4	011	$\vec{V_4} = \frac{2}{3} V_D e^{j3\pi/3}$	
V5	001	$\vec{V_5} = \frac{2}{3} V_D e^{j4\pi/3}$	
V6	101	$\vec{V_6} = \frac{2}{3} V_D e^{j5\pi/3}$	
V7	111	$\vec{V_7} = 0$	Zero Vector

Graphical way

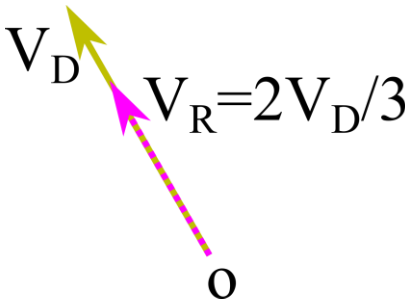
100 Condition



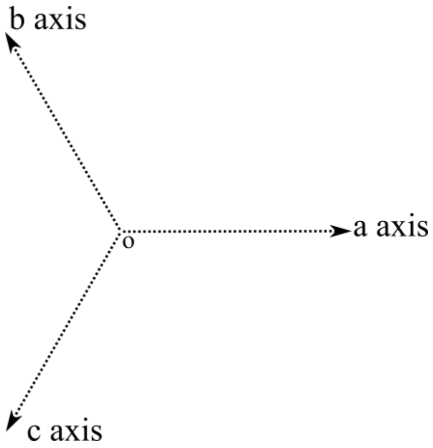
110 Condition



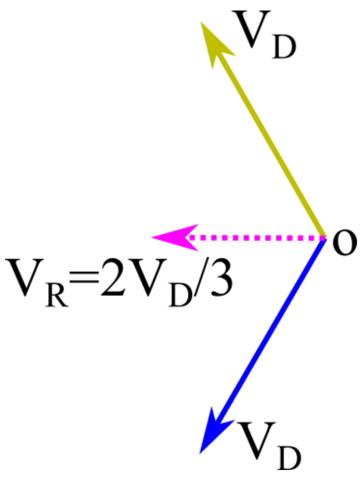
010 Condition



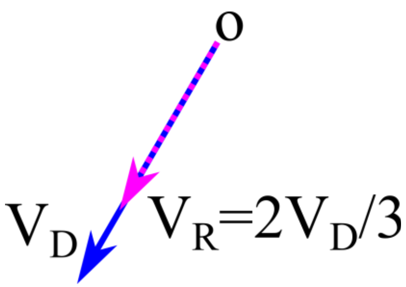
- The space vectors can be obtained also from a graphical method.



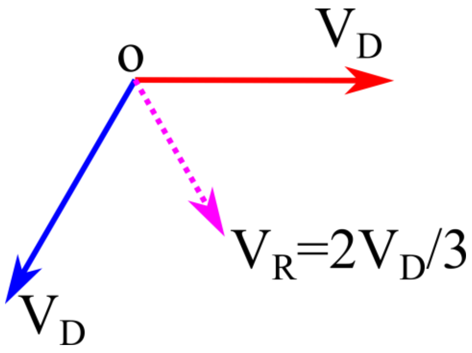
011 Condition



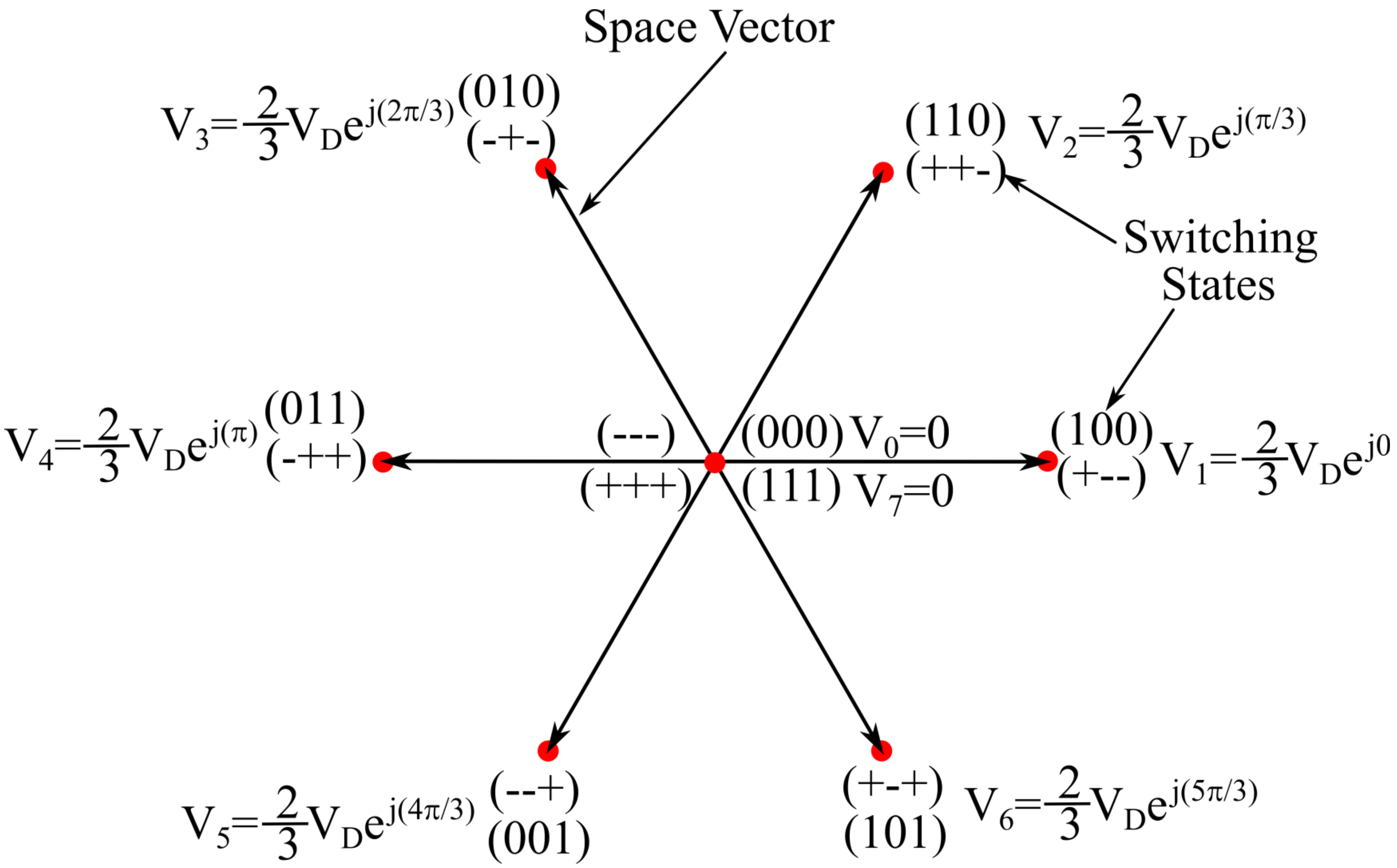
001 Condition



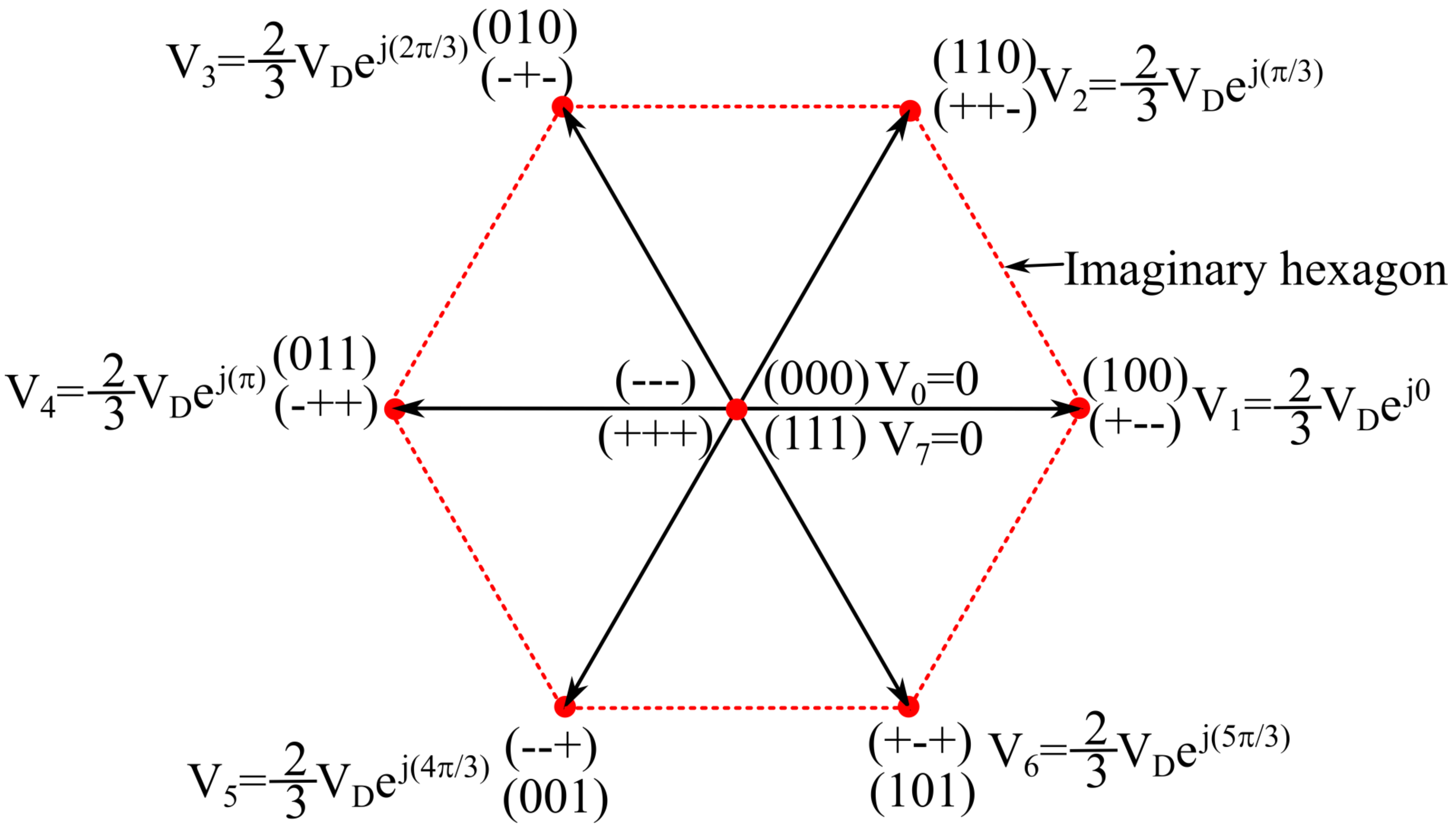
101 Condition



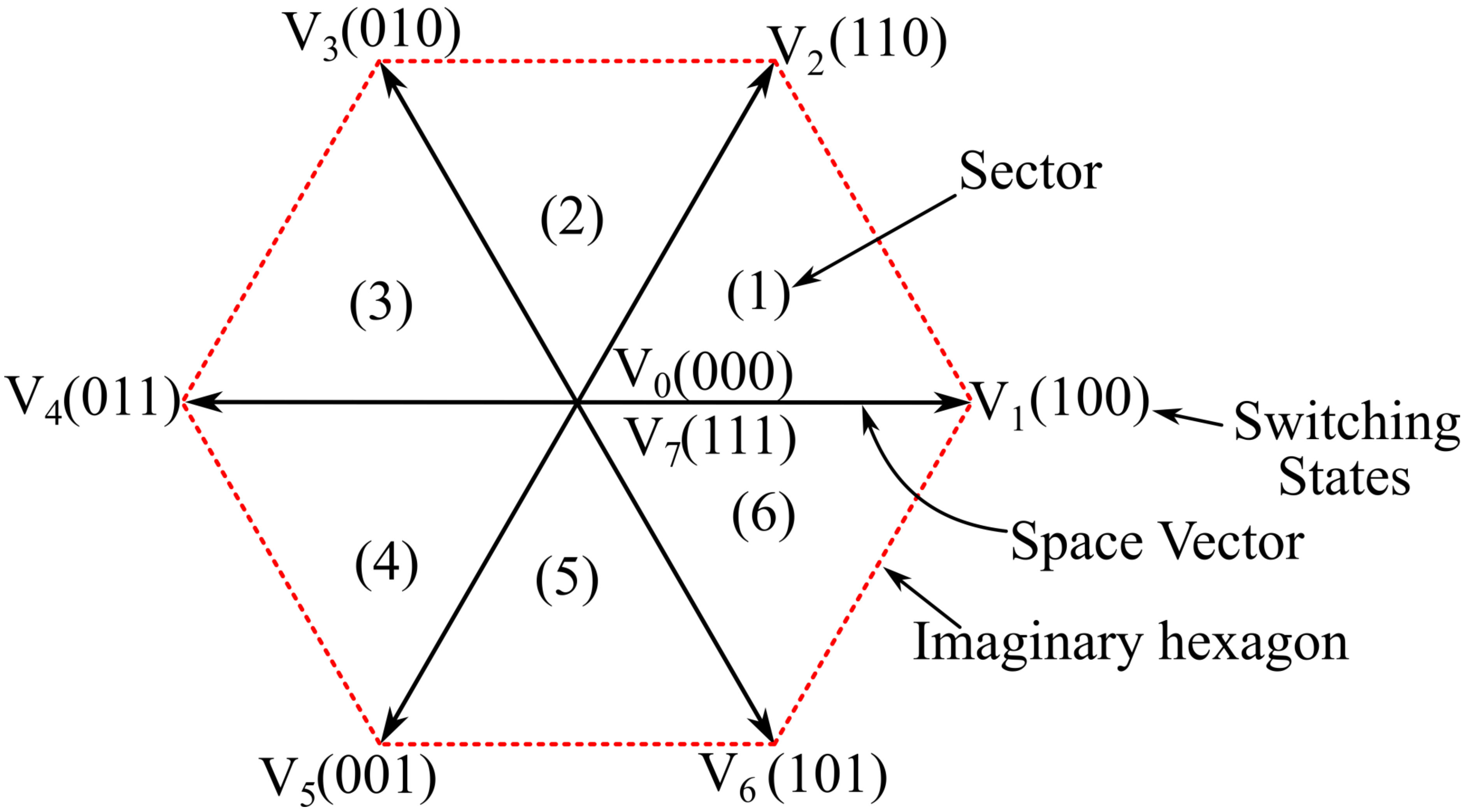
Eight space vectors



Boundary of space vector diagram



Sectors in space vector diagram

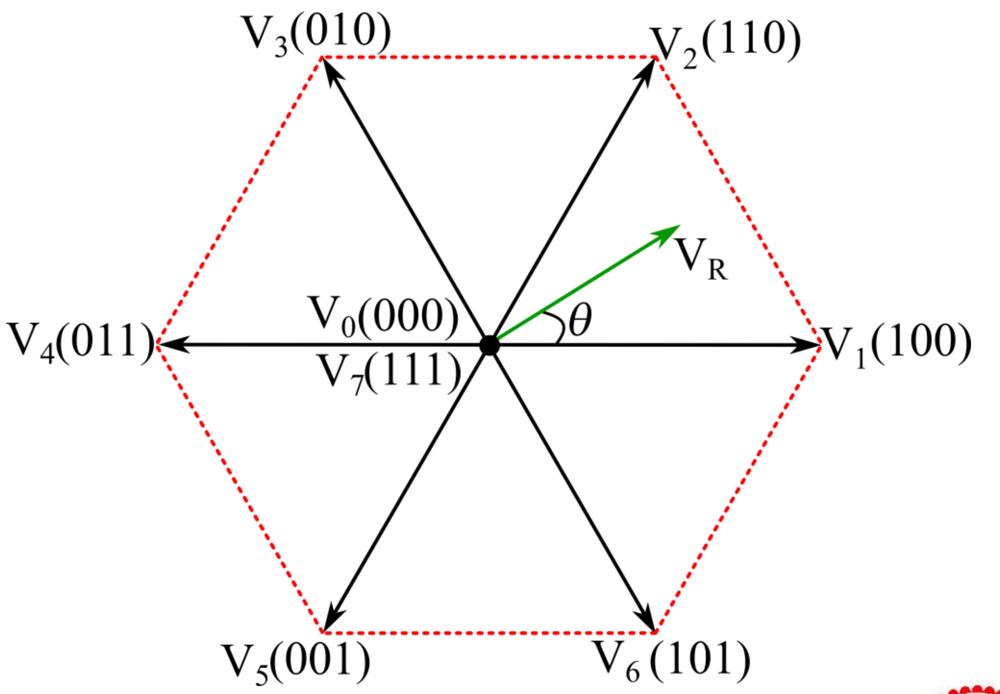


Space Vector PWM

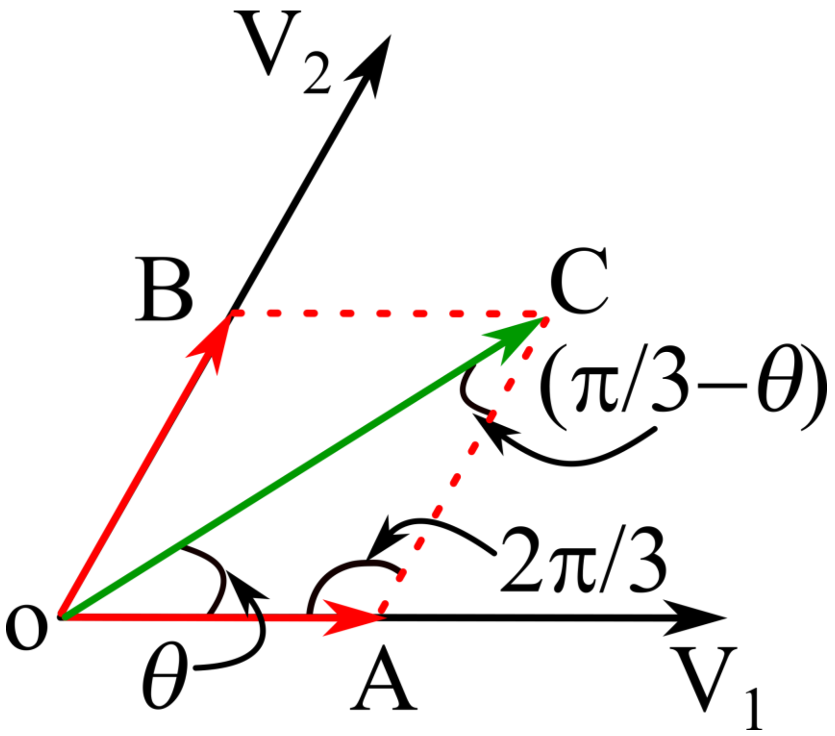
- How to switch the eight vectors so that the correct voltage is impressed on the load?
- Space vector PWM is an extension of sine triangle PWM. Here the PWM is done by using space vectors.

Space Vector PWM

- The space vectors are switched for certain duration of time in a cycle so as to produce the resultant vector.
- $\boldsymbol{V_R} T_S = \boldsymbol{V_1} T_1 + \boldsymbol{V_2} T_2 + \boldsymbol{V_0} T_0 = \boldsymbol{V_1} T_1 + \boldsymbol{V_2} T_2 + \boldsymbol{V_0} T_{01} + \boldsymbol{V_0} T_{07}$
- $T_S = T_1 + T_2 + T_0$
- In space vector PWM, $T_{01} = T_{07} = T_0/2$



Mathematical expression of timings



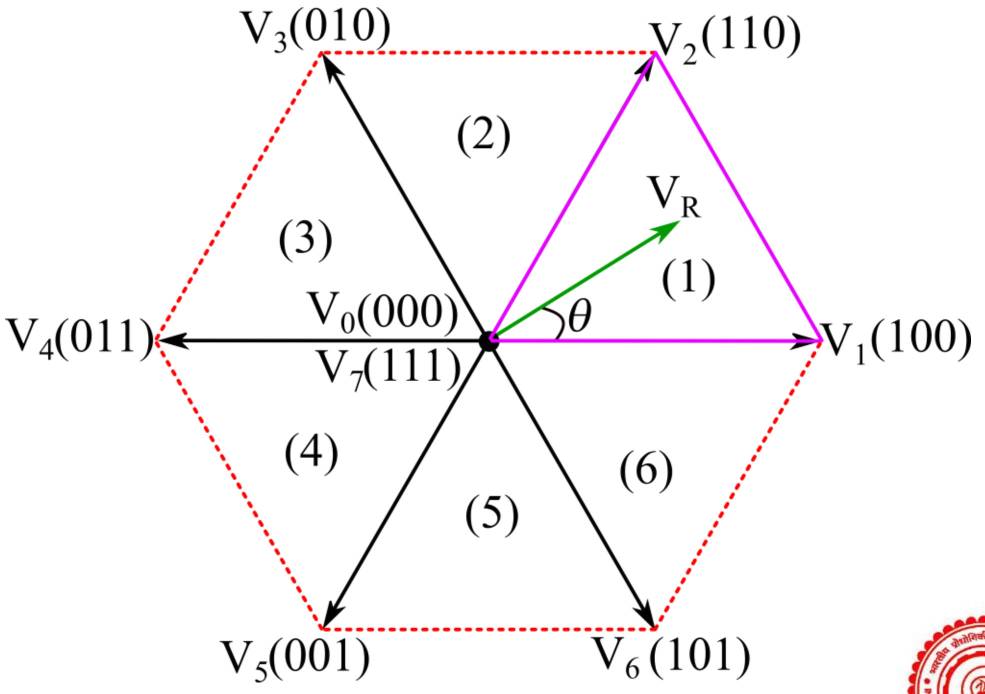
- $\frac{OA}{\sin(\frac{\pi}{3}-\theta)} = \frac{OB}{\sin(\theta)} = \frac{OC}{\sin(\frac{2\pi}{3})}$
- $\frac{V_1 T_1}{\sin(\frac{\pi}{3}-\theta)} = \frac{V_2 T_2}{\sin(\theta)} = \frac{V_R T_S}{\sin(\frac{2\pi}{3})}$
- $T_1 = \sin(\frac{\pi}{3} - \theta)\frac{V_R}{V_1}\frac{2}{\sqrt{3}}T_S = \sin(\frac{\pi}{3} - \theta)\frac{V_R}{V_D}\sqrt{3}T_S$
- $T_2 = \sin\theta\frac{V_R}{V_1}\frac{2}{\sqrt{3}}T_S = \sin\theta\frac{V_R}{V_D}\sqrt{3}T_S$
- $T_0 = T_S - T_1 - T_2$
- What happens at $\theta = 0$, and V_R=2/3 V_D?

Zero vector

- Usually the zero vectors are kept equal. This gives the best harmonic performance.
- $T_0 = T_S - T_1 - T_2$ is divided into equal parts of $T_0/2$ at the beginning and end of the cycle i.e. $T_{01} = T_{07} = \frac{T_0}{2}$
- For special switching sequences (e.g. discontinuous PWM), the division is made not equal.

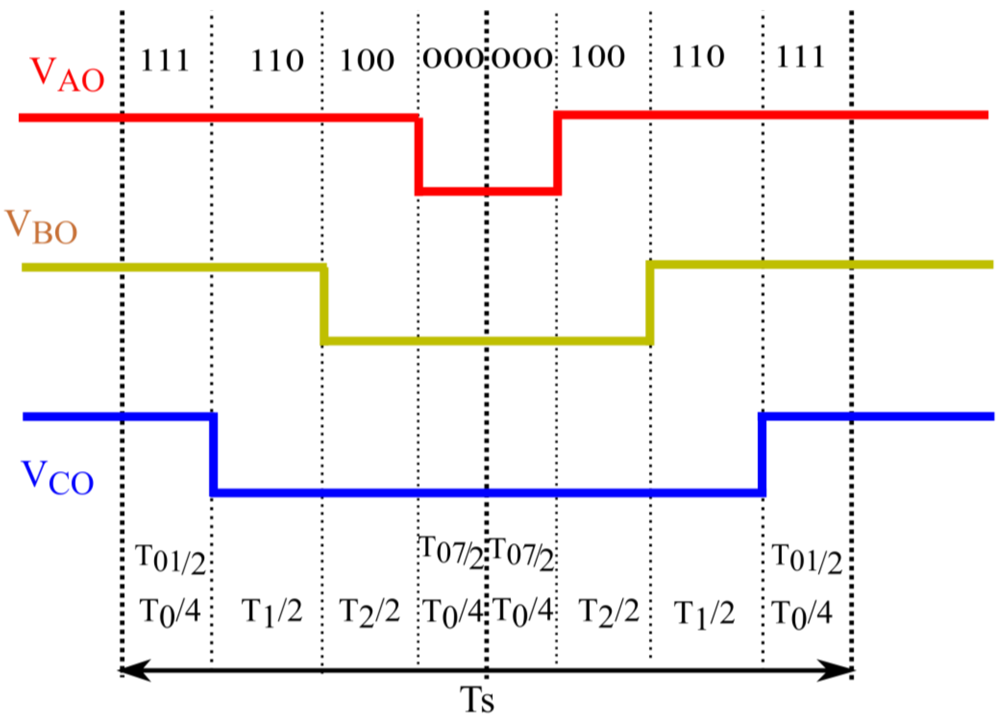
Example of switching

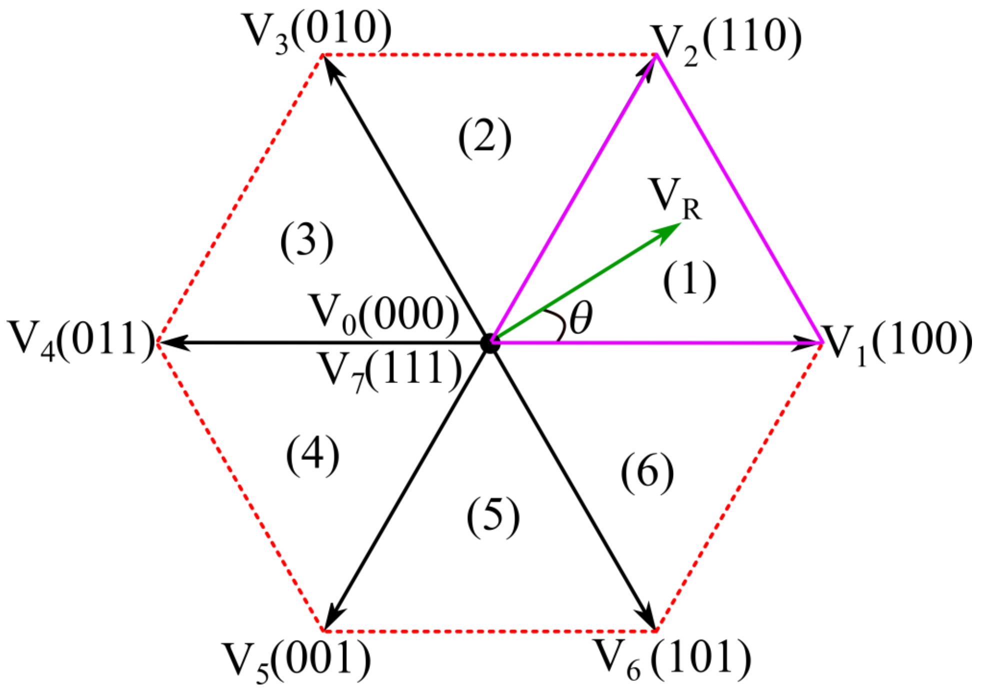
- For example, we can switch in a switching cycle T_s: 111 for T_{01} time period, 110 for T_1 time period, 100 for T_2 time period and 000 for T_{07} time period. This will realize the reference vector ($\boldsymbol{V_R}$) in the switching cycle T_s.
- $\boldsymbol{V_R} T_S = \boldsymbol{V_1} T_1 + \boldsymbol{V_2} T_2 + \boldsymbol{V_0} T_0 = \boldsymbol{V_1} T_1 + \boldsymbol{V_2} T_2 + \boldsymbol{V_0} T_{01} + \boldsymbol{V_0} T_{07}$



Example of switching

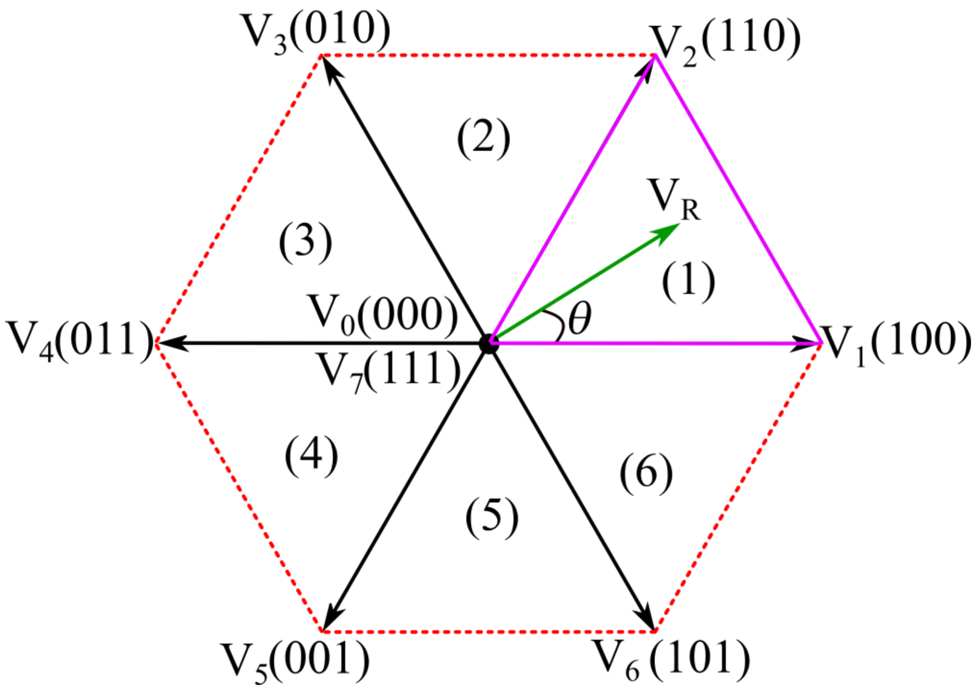
- The instantaneous pole voltages can be seen from the diagram. The switching sequence is 111-110-100-000-100-110-111 and so on in sector 1.
- The sequence ensures minimum switching.



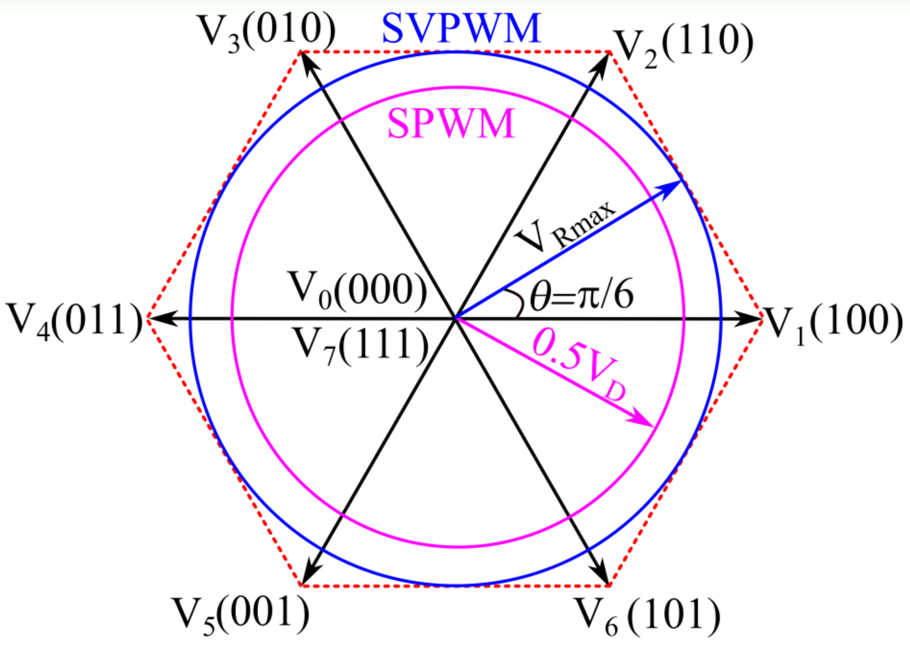


Example of switching

- Similarly, 111-110-010-000-010-110-111 and so on in sector 2.



What is the maximum voltage?

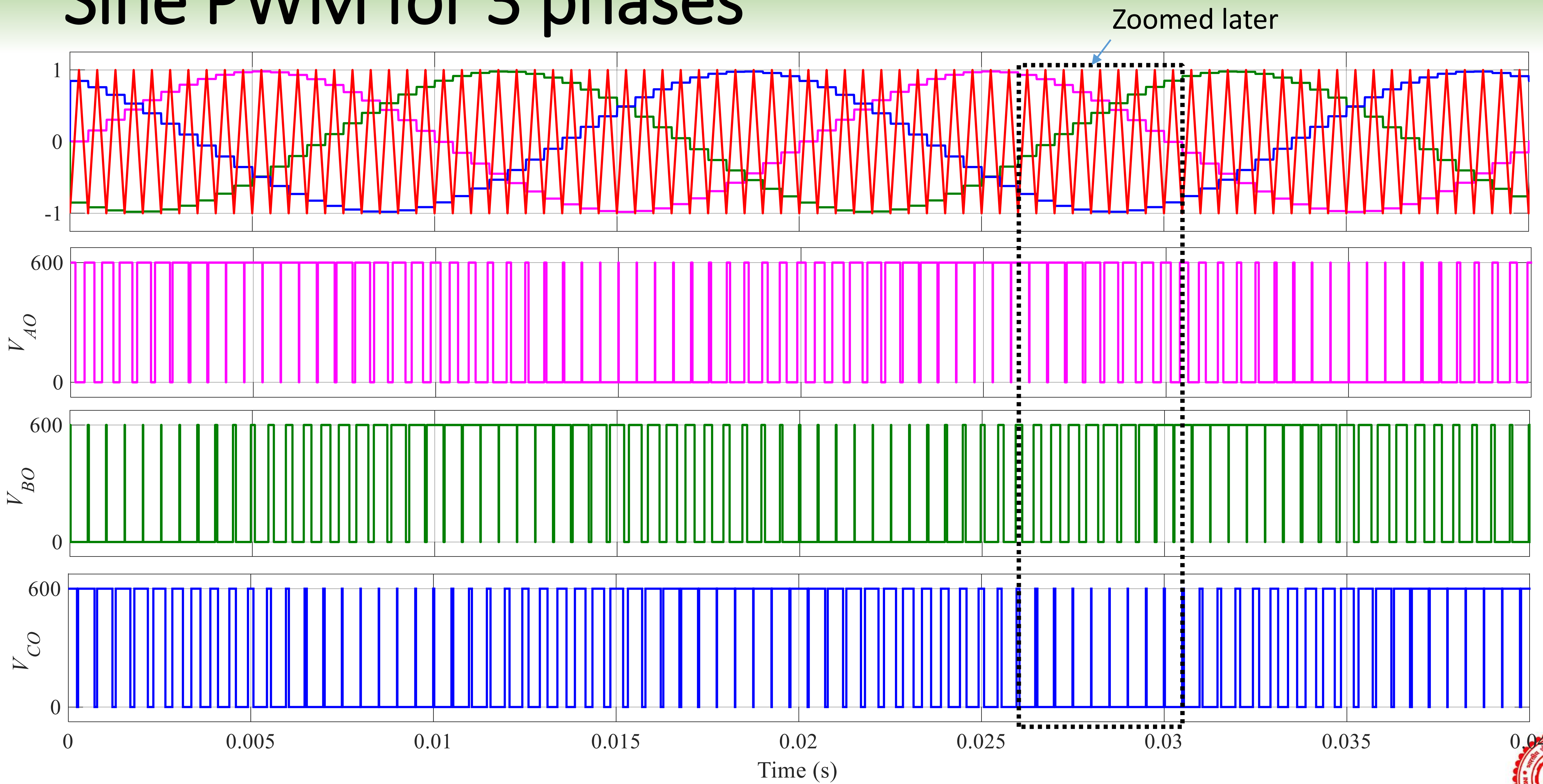


- The maximum voltage is obtained in linear modulation when the inscribed circle touches the hexagon.
- $V_{Rmax} = \frac{2}{3} V_D \cos\frac{\pi}{6}$=0.577 V_D
- In sine-PWM the peak AC voltage that was obtained was 0.5 V_D.

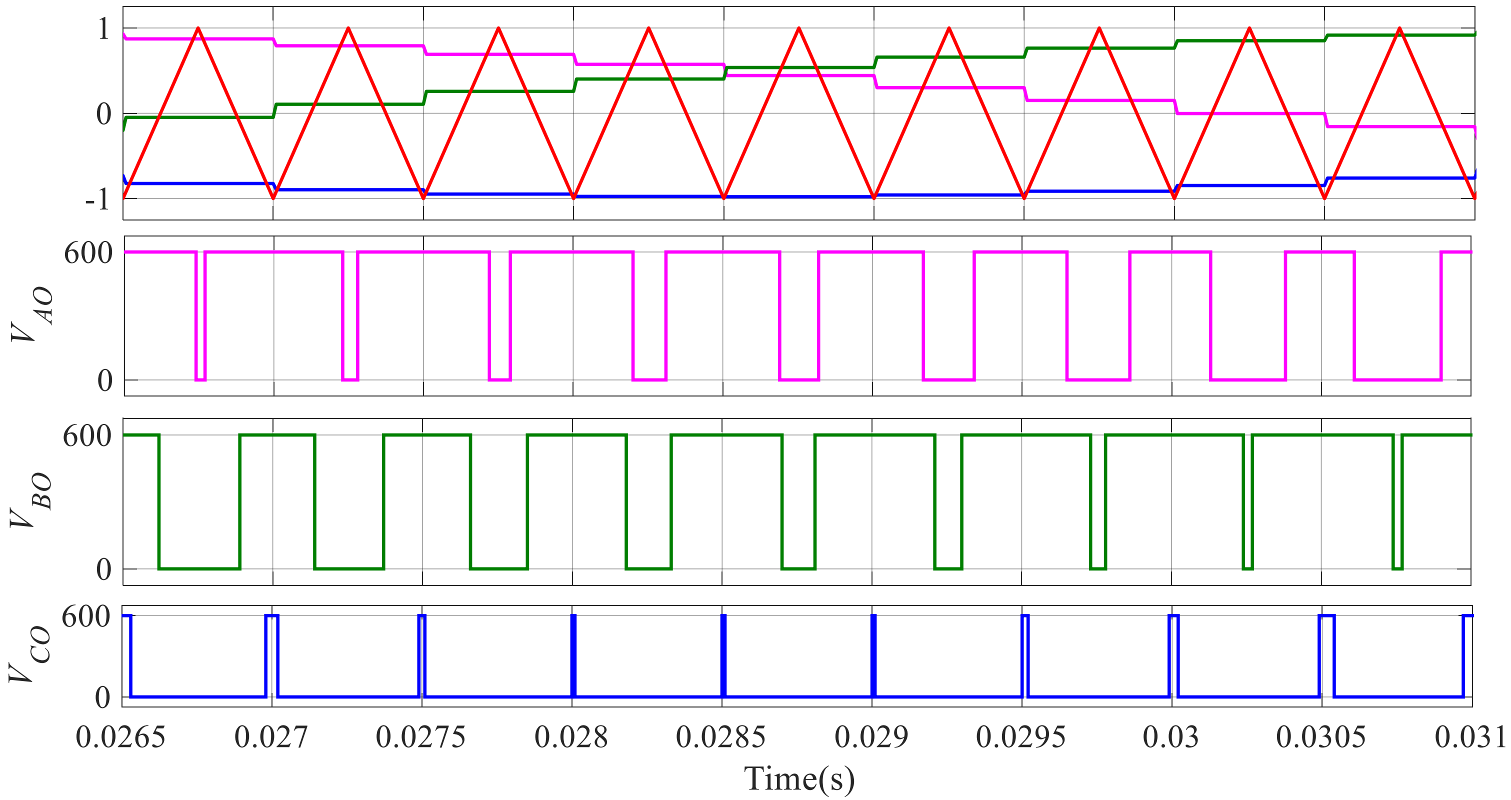
How to realize using carriers?

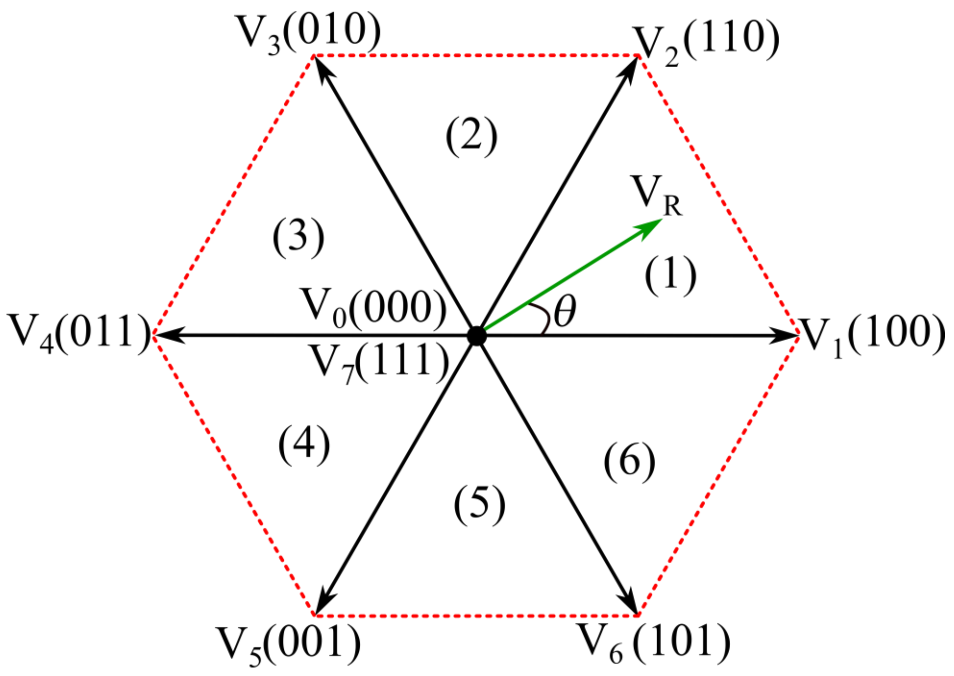
- The SVPWM technique discussed so far involves substantial calculation, sector identification etc.
- It can be done very easily using carriers where no calculation, sector identification or switching sequence design is required.
- In order to realize SVPWM through carriers, we can observe the sine PWM more in details.

Sine PWM for 3 phases



Zoomed view in sine PWM

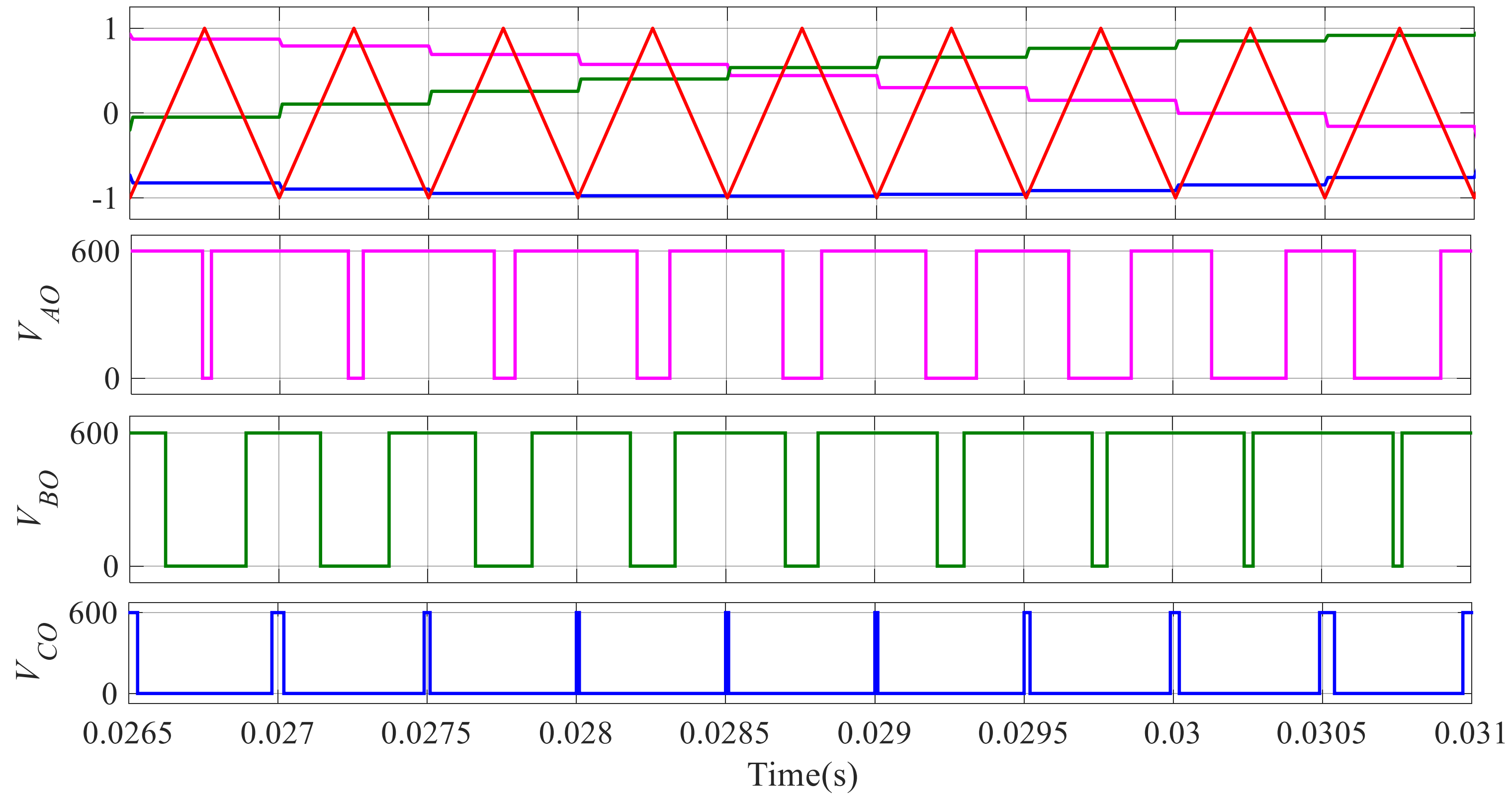


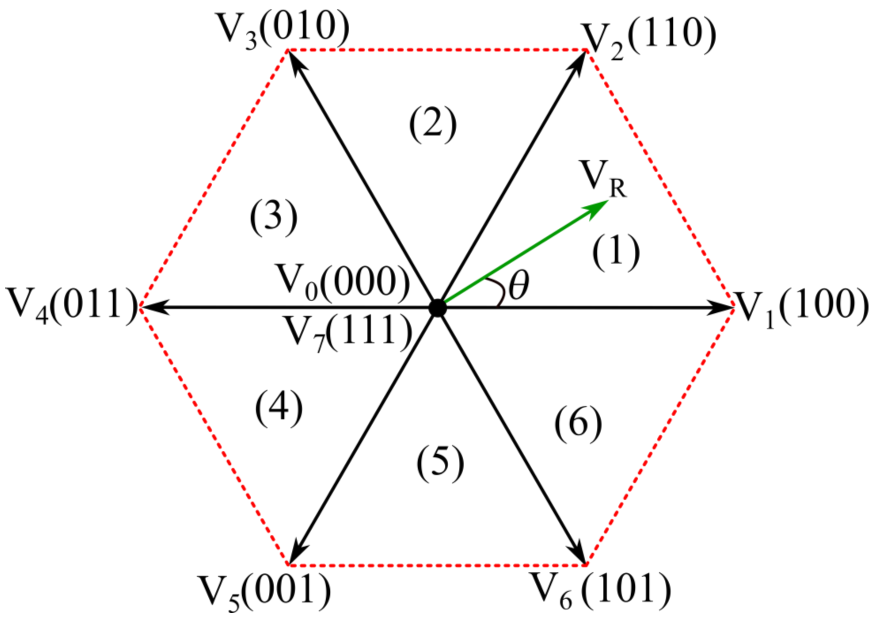


- What is the pattern of switching?

Zoomed view in sine PWM

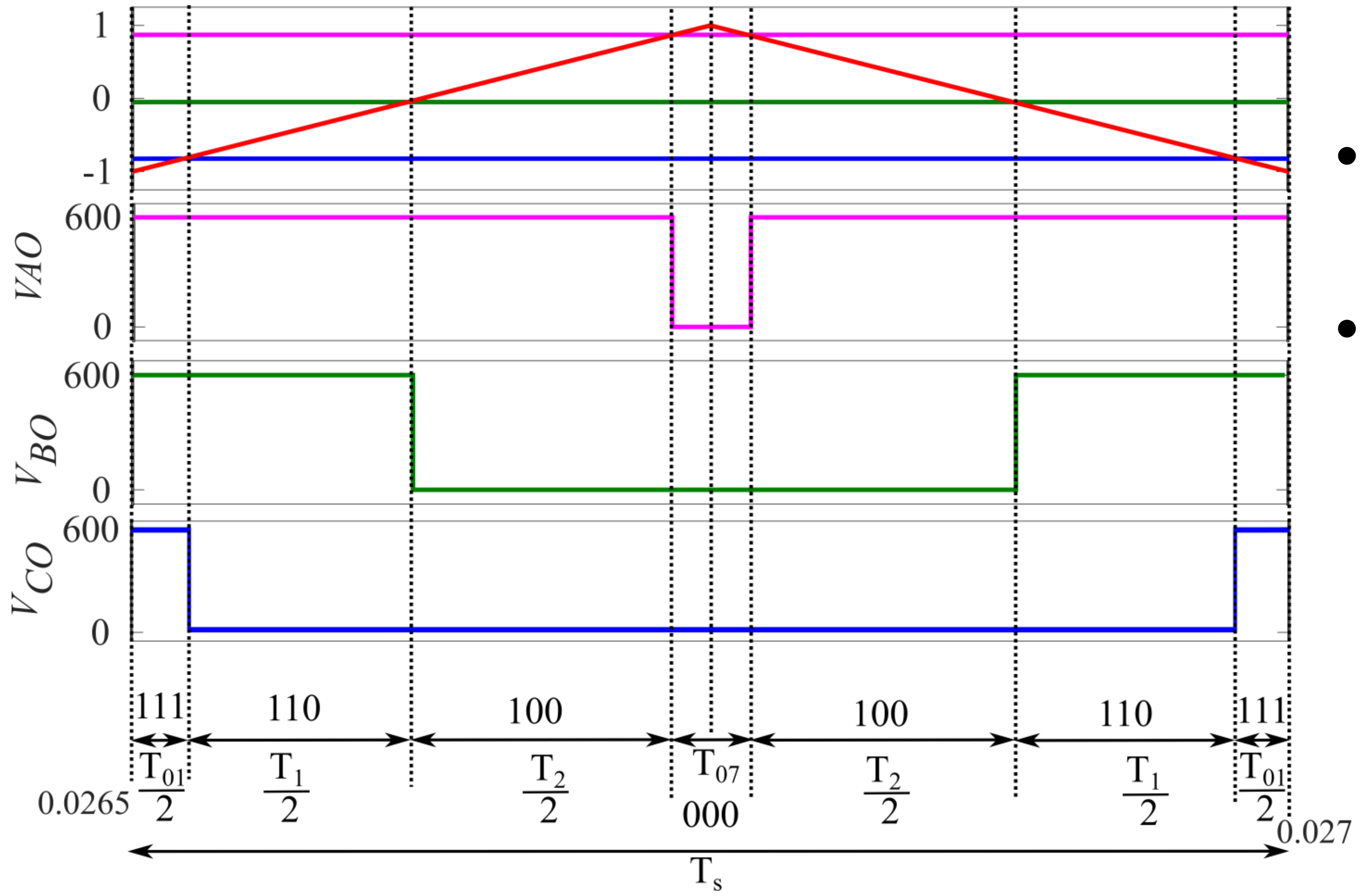
SECTOR 1 SECTOR 2



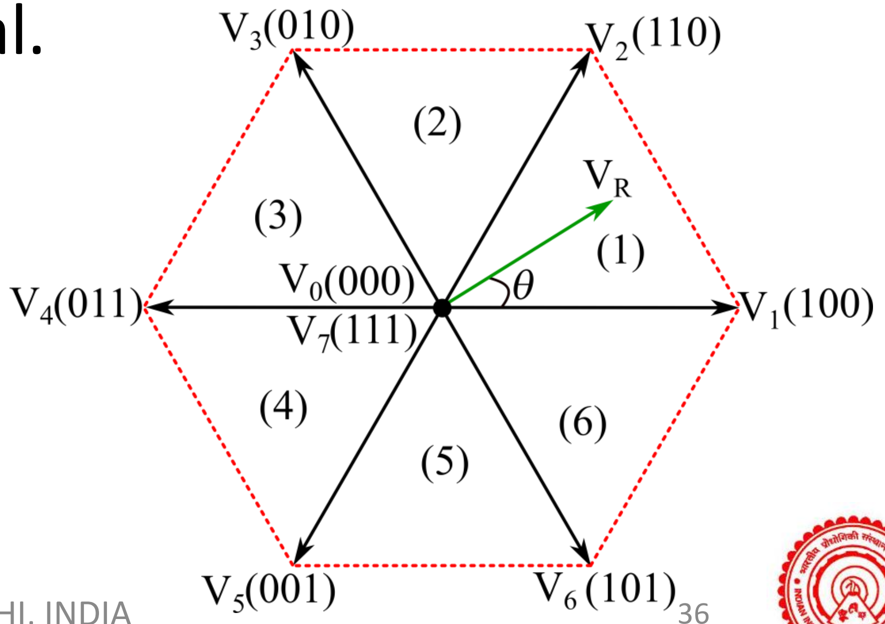


- 111-110-100-000-100-110-111 and so on in sector 1.
- 111-110-010-000-010-110-111 and so on in sector 2.

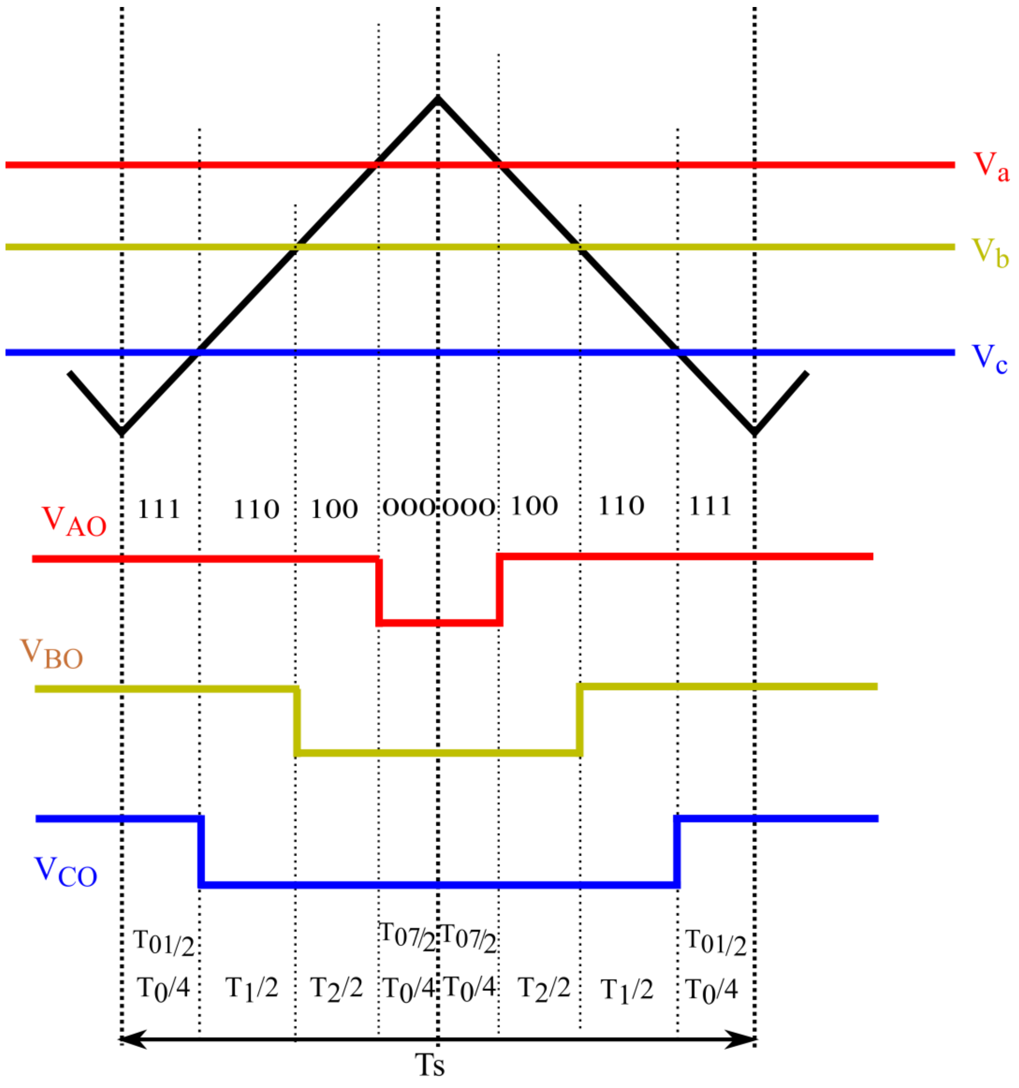
Zoomed view in one carrier in sine PWM



- Let us zoom further into one carrier.
- We observe that the two zero vector periods are not equal.
- In Sine PWM, T_{01} time period and T_{07} time period are not always equal.

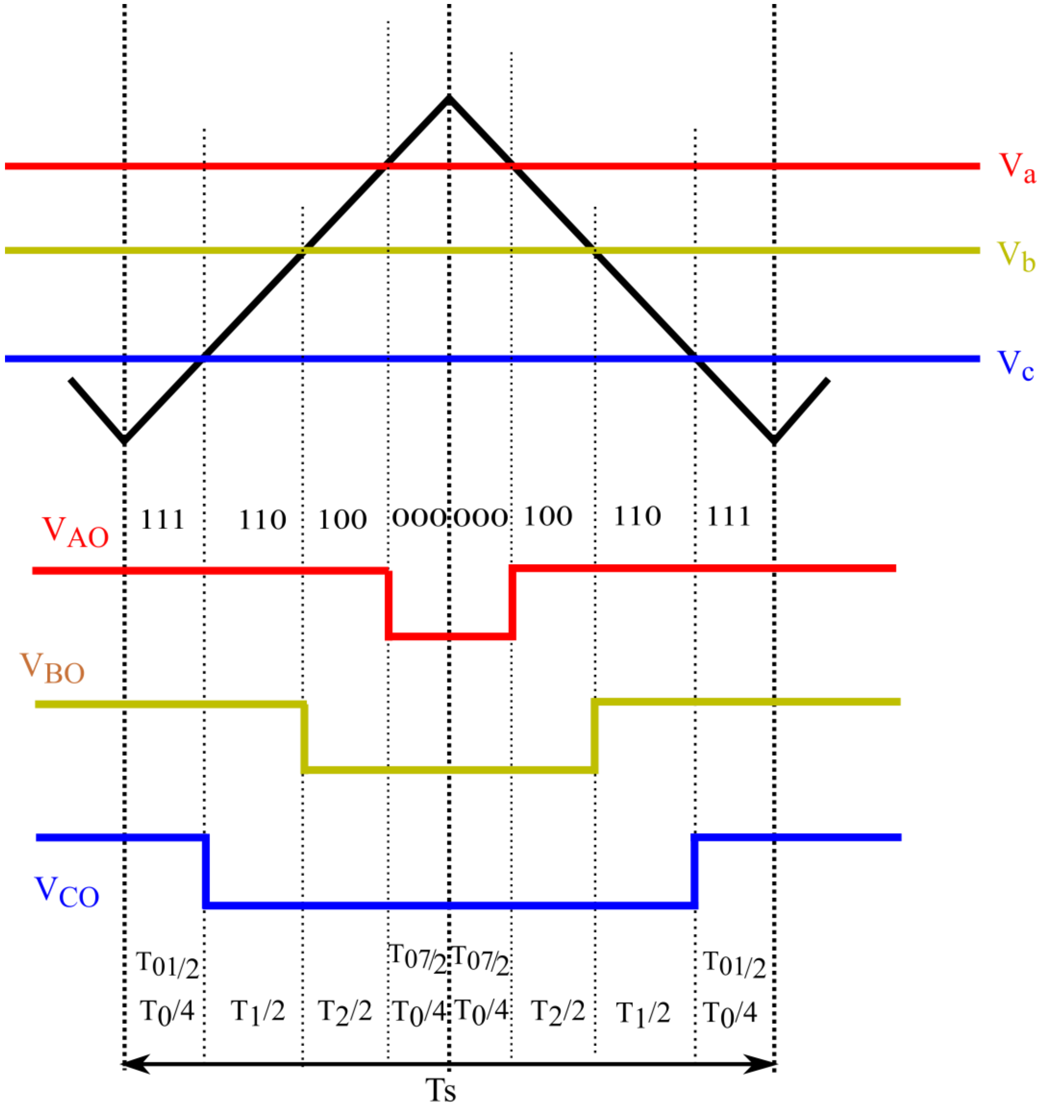


Switching sequence in space vector PWM



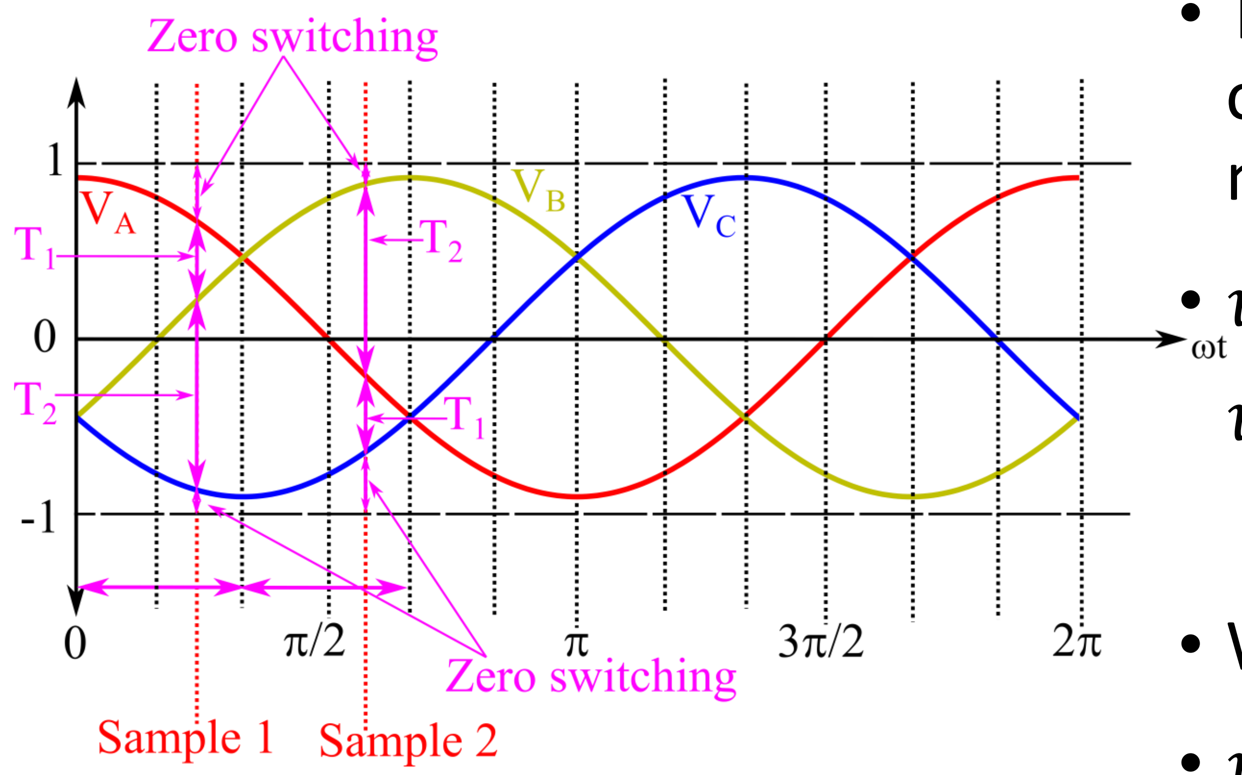
- The switching sequence here in one carrier period is 111-110-100-000-100-110-111.
- This is same as sine PWM, however the two zero periods are equal.

Switching sequence in space vector PWM



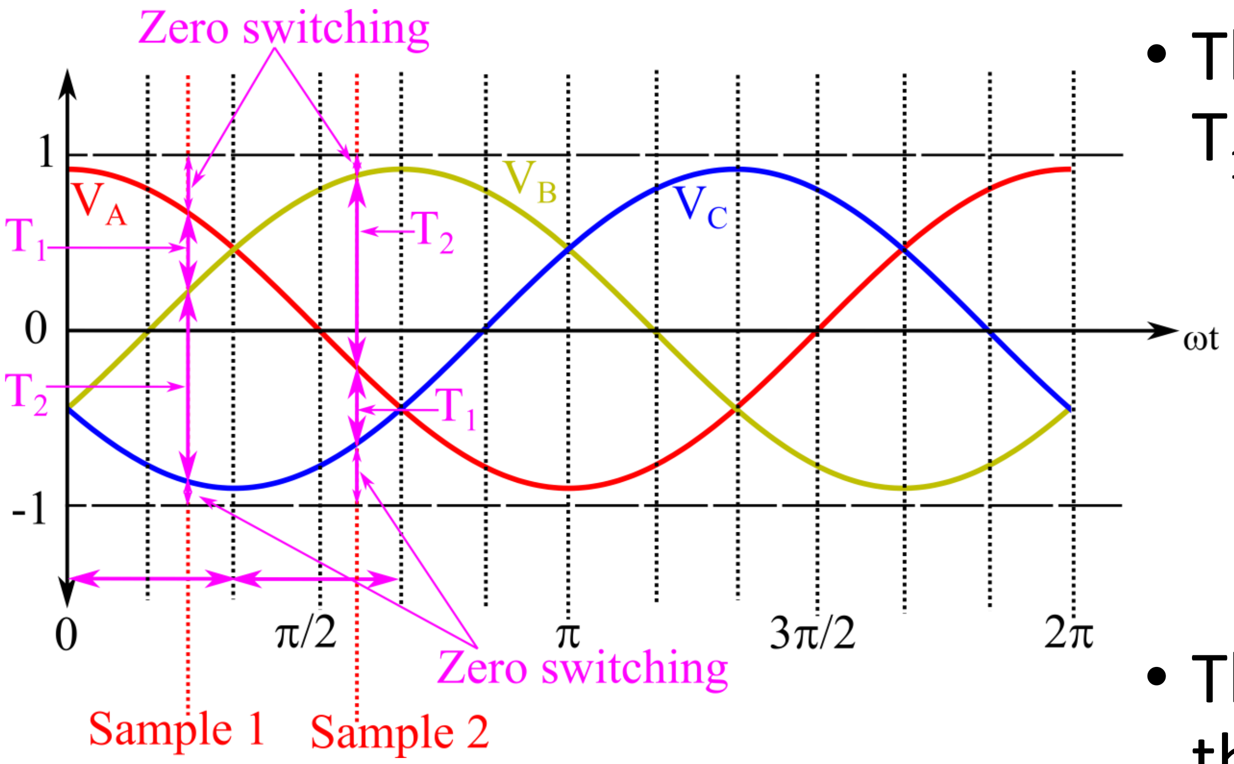
- Minimum switching is ensured.
- Switching frequency is same as carrier frequency.
- Thus space vector PWM is an extension of sine PWM, and can also be realized using carriers.

Extension of sine PWM



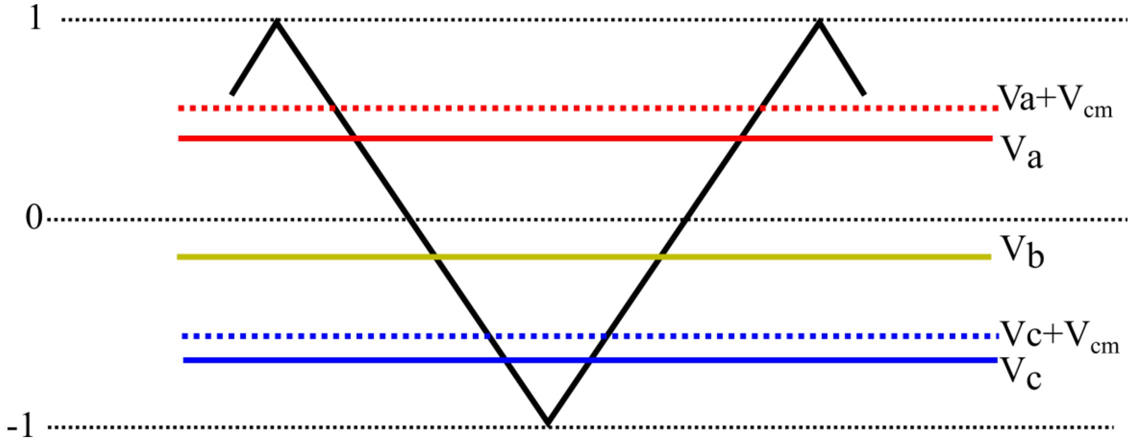
- The space vector PWM is an extension of sine PWM by addition of a common mode voltage.
- $v_a = V_m \cos\theta, v_b = V_m \cos(\theta - \frac{2\pi}{3}),$ $v_c = V_m \cos(\theta - \frac{4\pi}{3})$
- What are the line voltages v_{ab} and v_{bc} ?
- $v_{ab} = \sqrt{3}\, V_m \sin(\frac{\pi}{3} - \theta)$
- $v_{bc} = \sqrt{3}\, V_m \sin\theta$

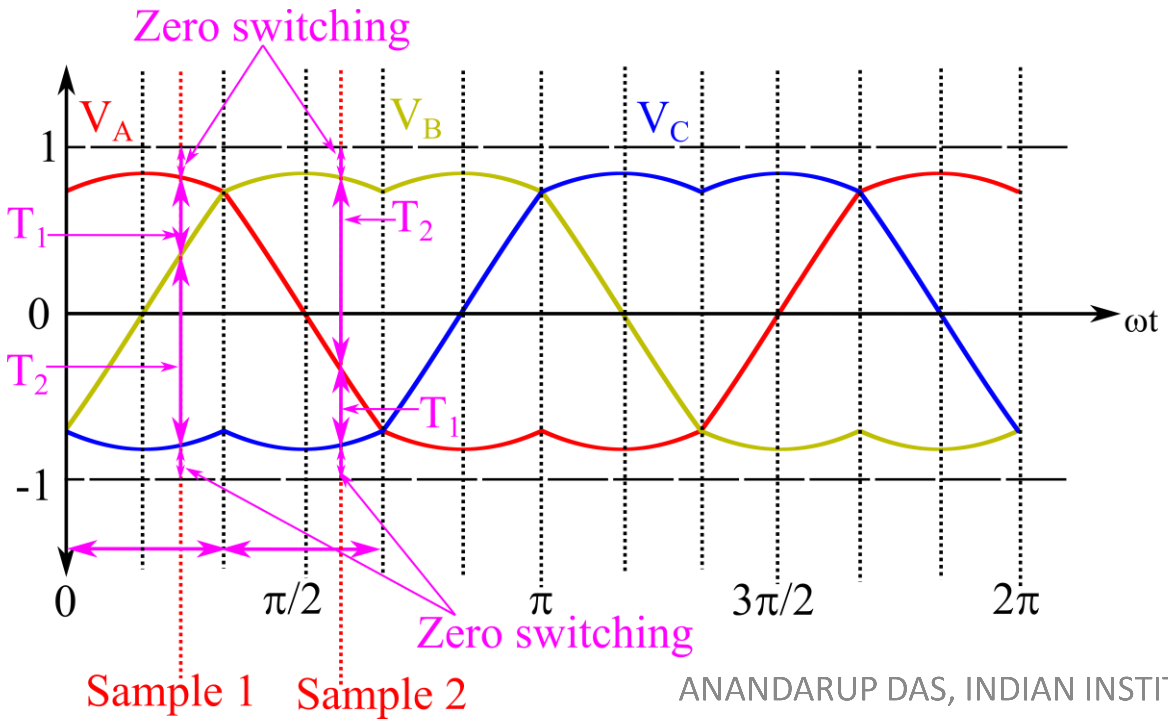
Extension of sine PWM



- The line voltage expressions follow the T_1 and T_2 expressions.
  - $v_{ab} = \sqrt{3}\, V_m \sin(\frac{\pi}{3} - \theta)$
  - $T_1 = \sqrt{3}\frac{V_R}{V_D} T_S \sin(\frac{\pi}{3} - \theta)$
  - $v_{bc} = \sqrt{3}\, V_m \sin\theta$
  - $T_2 = \sqrt{3}\frac{V_R}{V_D} T_S \sin\theta$
- The active vectors are represented by the line voltages.
- What about the zero vectors?

Common mode voltage



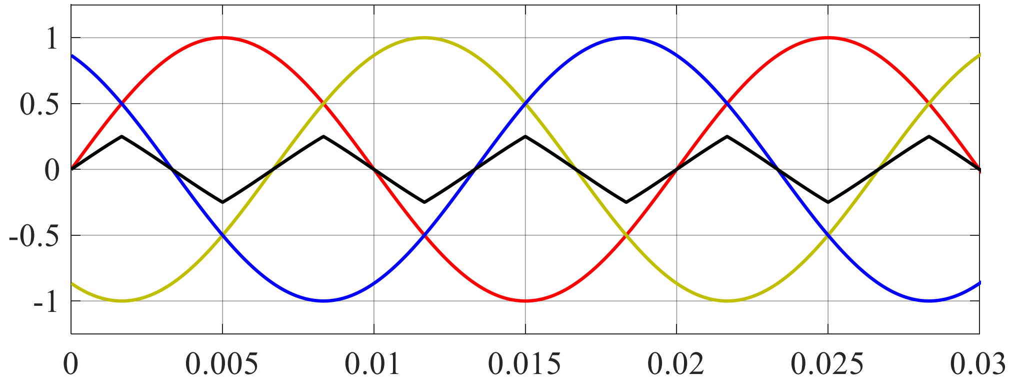


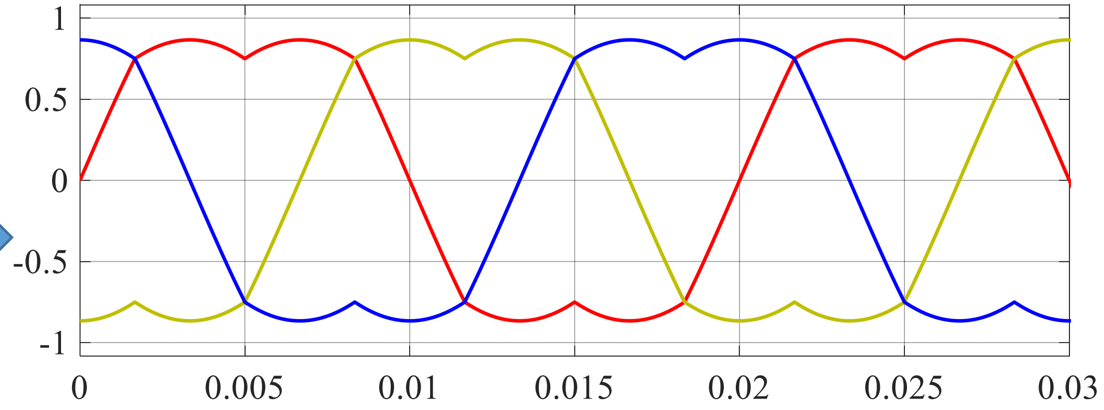
- To make the two zero vectors equal we have to add a common mode voltage.
- What should be its value?
- $1-(v_a+v_{cm}) = 1-v_c+v_{cm}$
- $v_{cm} = -\frac{v_a+v_c}{2}$
- In general, $v_{cm} = -\frac{v_{max}+v_{min}}{2}$

Waveforms

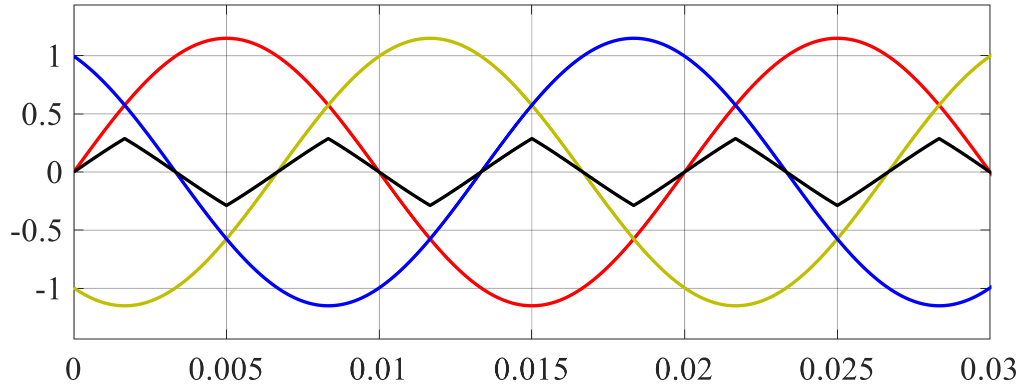
m=1

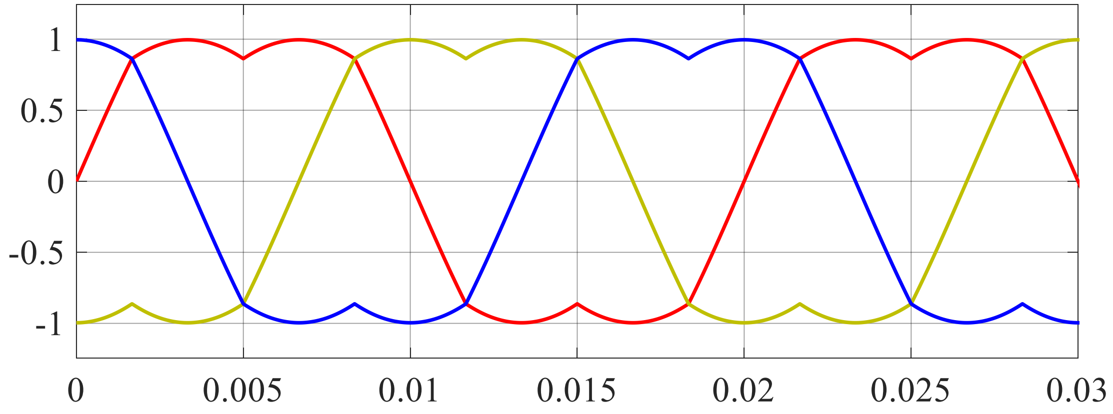
Resultant Waveform





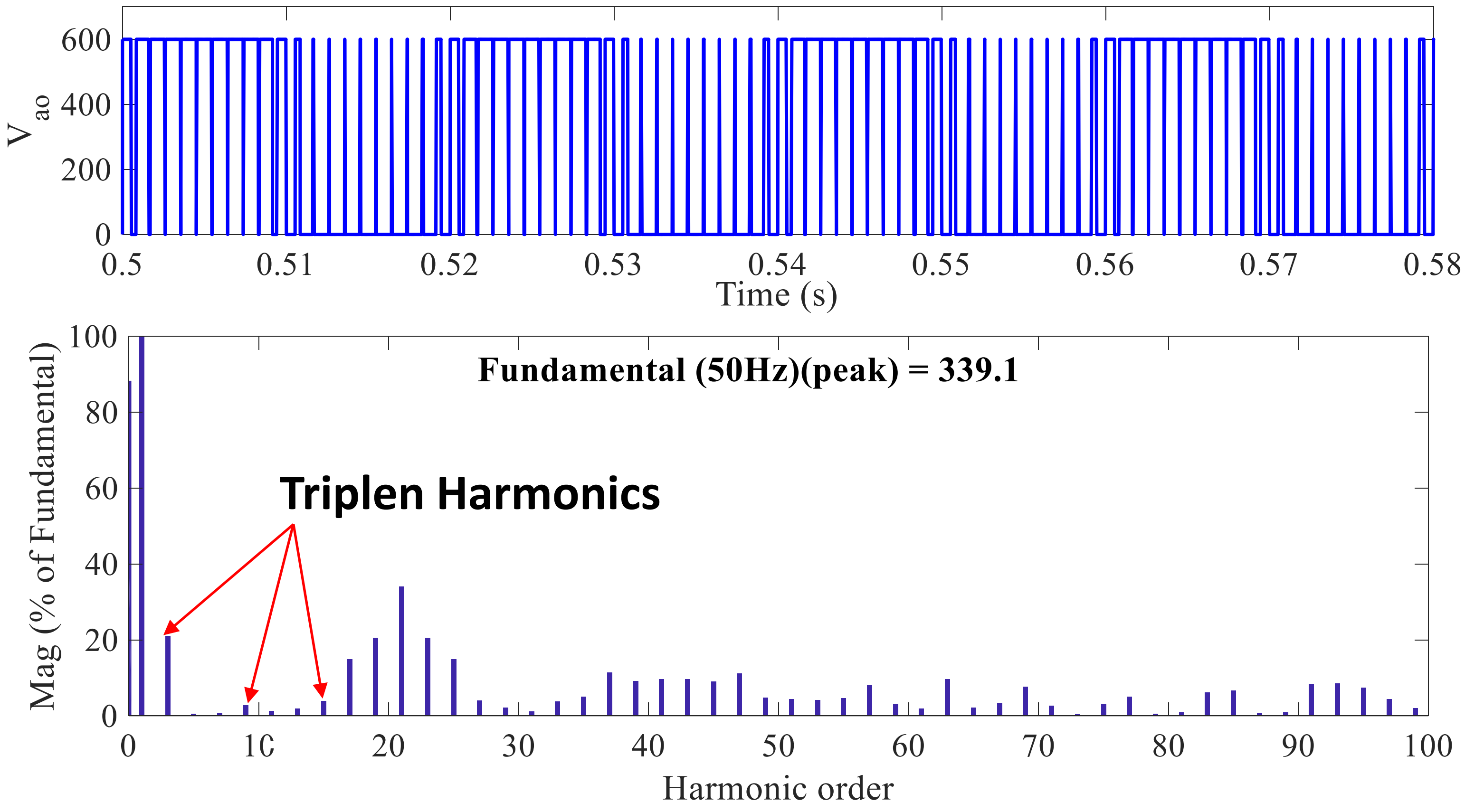
m=1.15





Simulation Waveforms

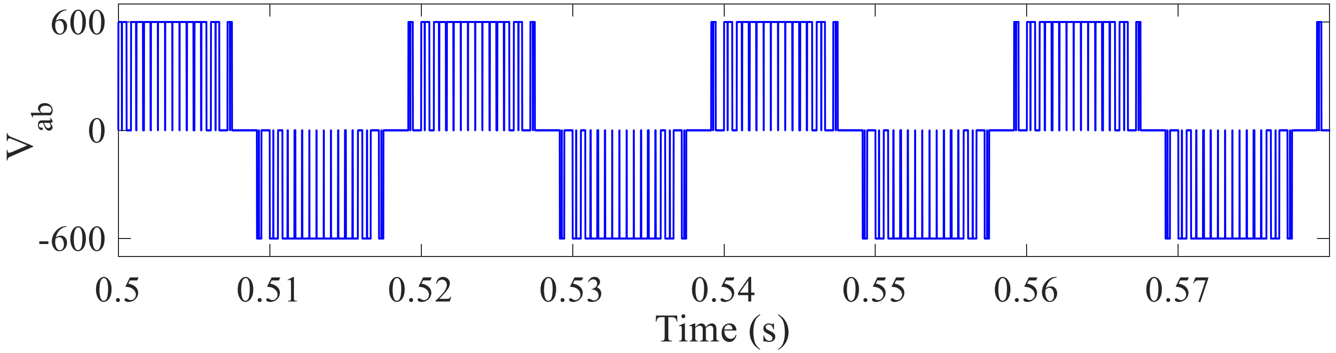
Pole Voltage

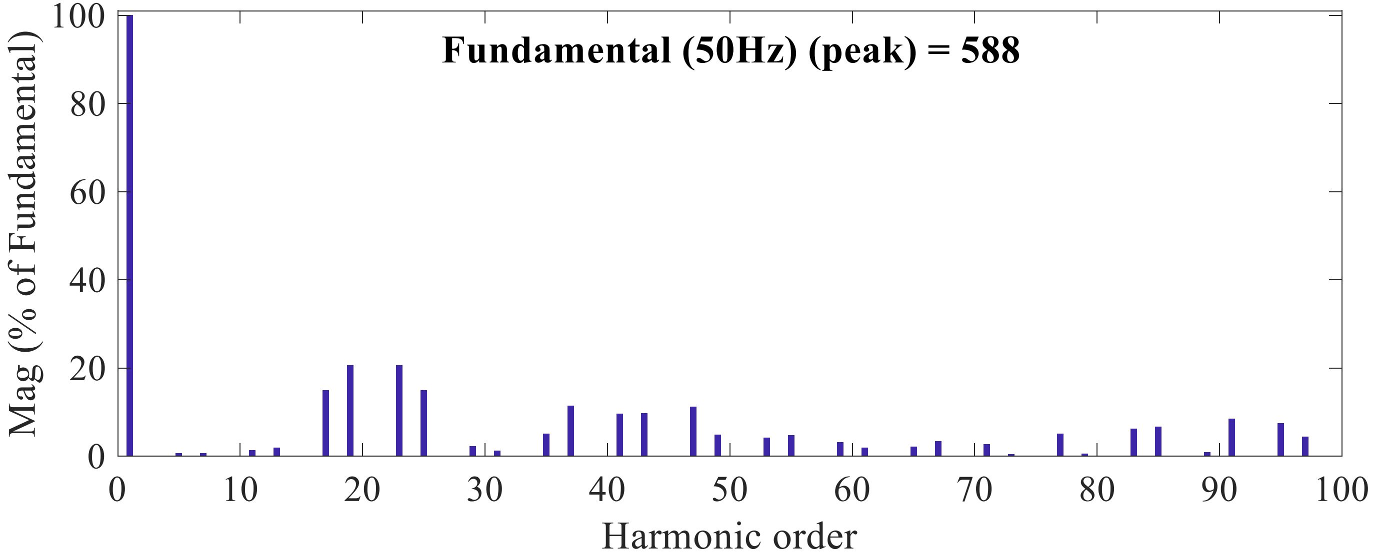


- V_{DC}=600V, the fundamental pole voltage is (1.154*600)*0.5*0.98=339.27 V
- mf =21, harmonics reside around *mf* , *2mf* , *3mf* …

Simulation Waveforms

Line voltage

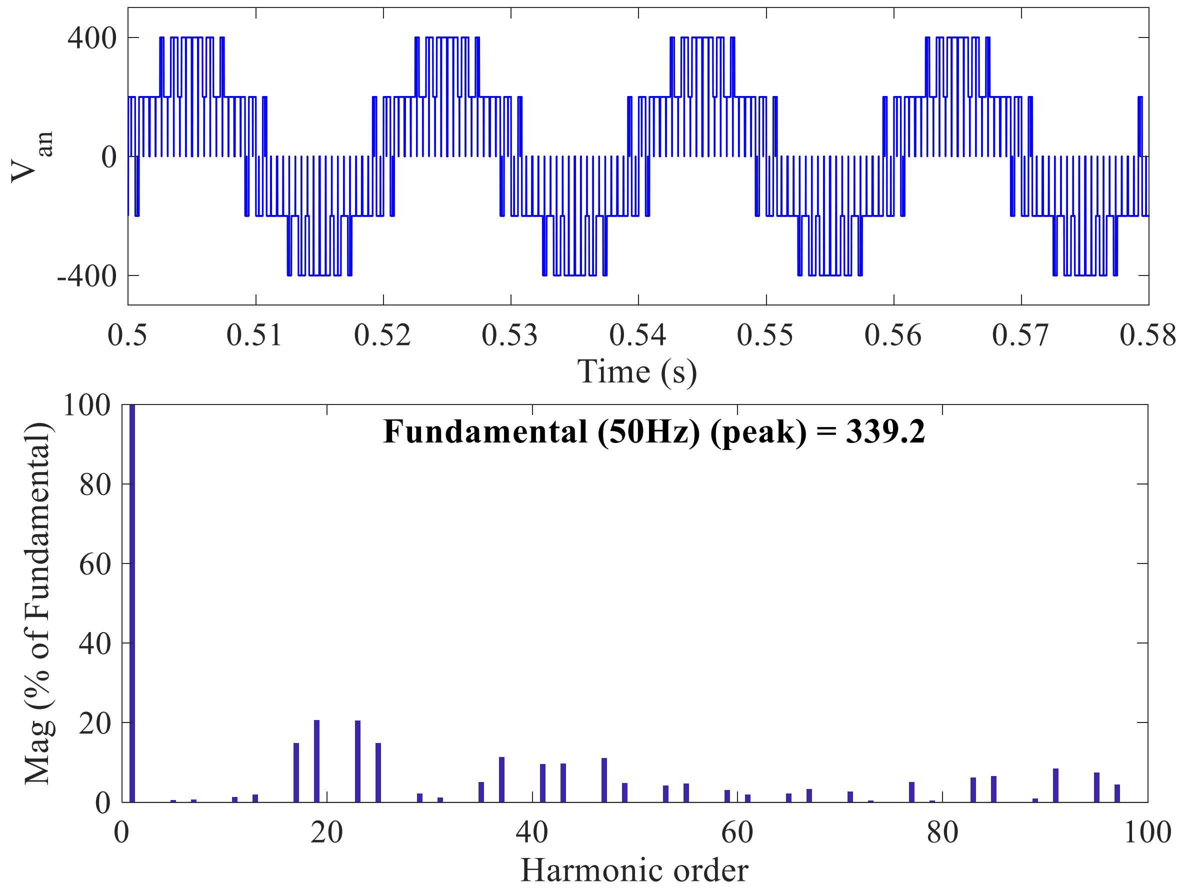




- V_{DC}=600V, the fundamental line voltage is (1.154*0.98*600)*0.5*1.732*=587.62 V
- mf =21, harmonics reside around *mf* , *2mf* , *3mf* …

Simulation Waveforms

Pole voltage



- The phase voltage does not contain any triplen harmonic, so the phase current will be absent from it.