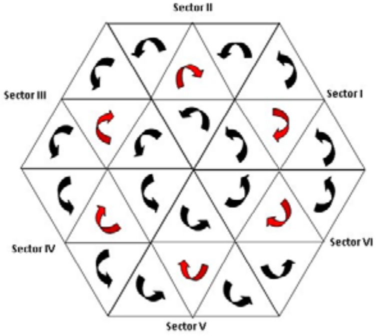
Chapter 4

Space Vector Modulation



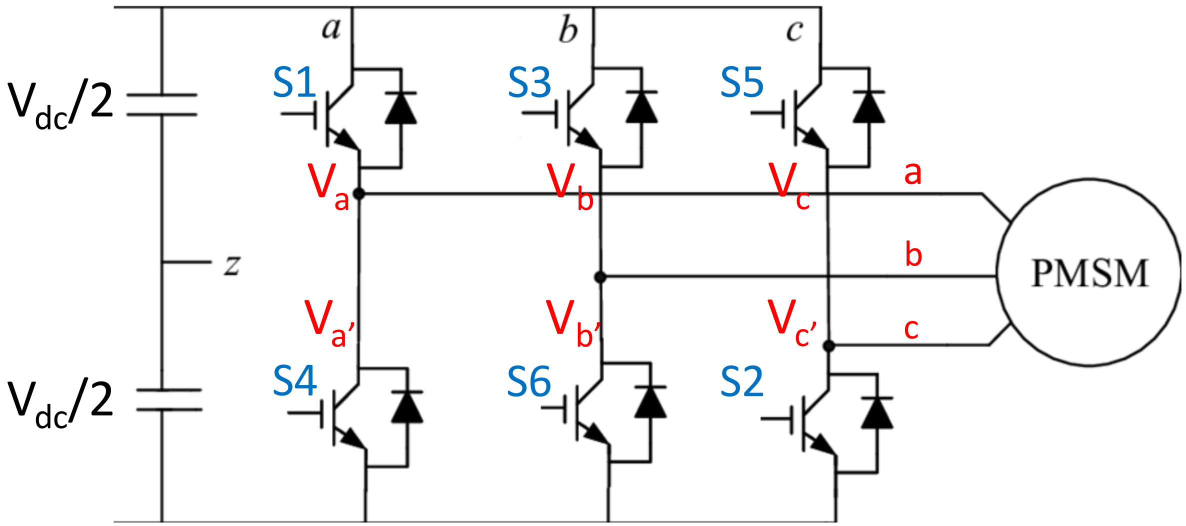
Professor Min-Fu Hsieh

Fall Semester - 2019

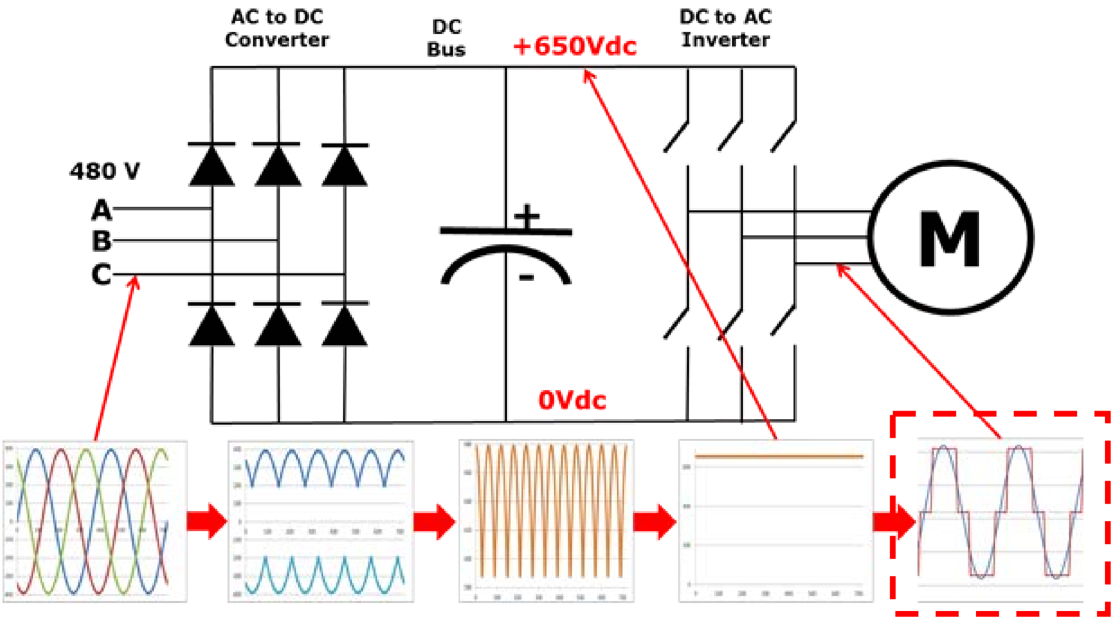
https://www.researchgate.net/publication/318930471_SIMULATION_AND_IMPLEMENTATION_OF_TWO-LEVEL_AND_THREE-LEVEL_INVERTERS_BY_MATLAB_AND_RT-LAB/figures?lo=1

Three Phase Inverter Control

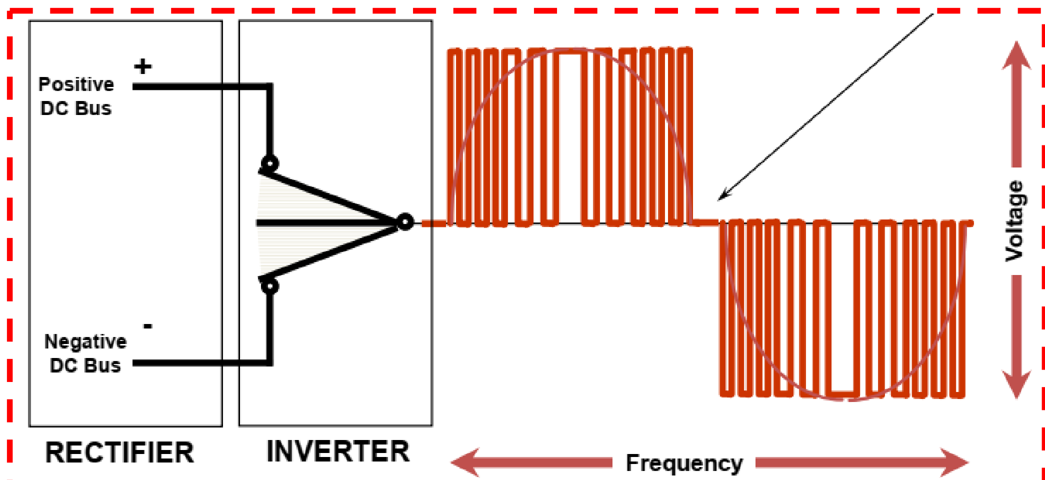
An inverter fed 3-phase AC motor with three phase windings a, b and c. The three phase voltages are applied by three pairs of semiconductor switches (S1 through S6) $V_a/V_{a'}$, $V_b/V_{b'}$ and $V_c/V_{c'}$ with amplitude, frequency and phase angle defined by microcontroller calculated pulse patterns. The inverter is fed by the DC link voltage V_{dc}



PWM in a AC Drive



How often you switch from positive pulses to negative pulses determines the frequency of the waveform



- https://www.vfds.com/blog/what-is-a-vfd

Pulse-Width Modulation (PWM)

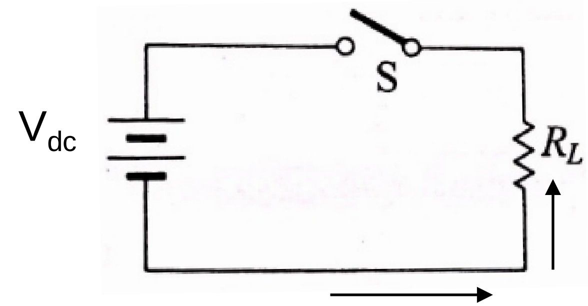
The basic principle of PWM can be stated that under constant supply voltage V_{dc}, when voltage is applied to the resistance load in pulses. The instantaneous output voltage v_0 is determined:

$$V_0 = \frac{T_{ON}}{T_{ON} + T_{OFF}} V_{dc}$$

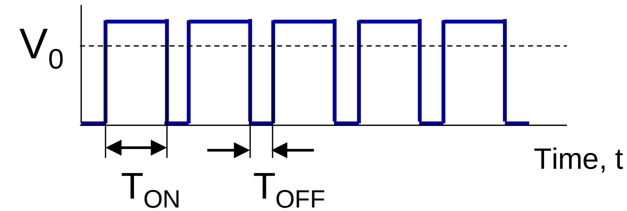
The voltage applied to the load can thus be changed using the time duty-cycle ratio

$$m = \frac{T_{ON}}{T_{ON} + T_{OFF}}$$

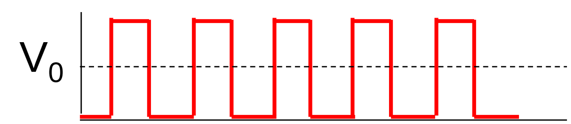
Pulses with fixed frequency and magnitude but variable width



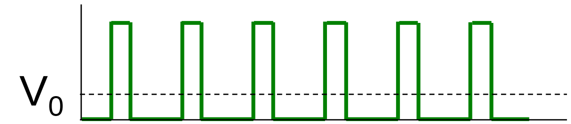
V_0 (Instantaneous voltage)



Large time ratio



Medium time ratio

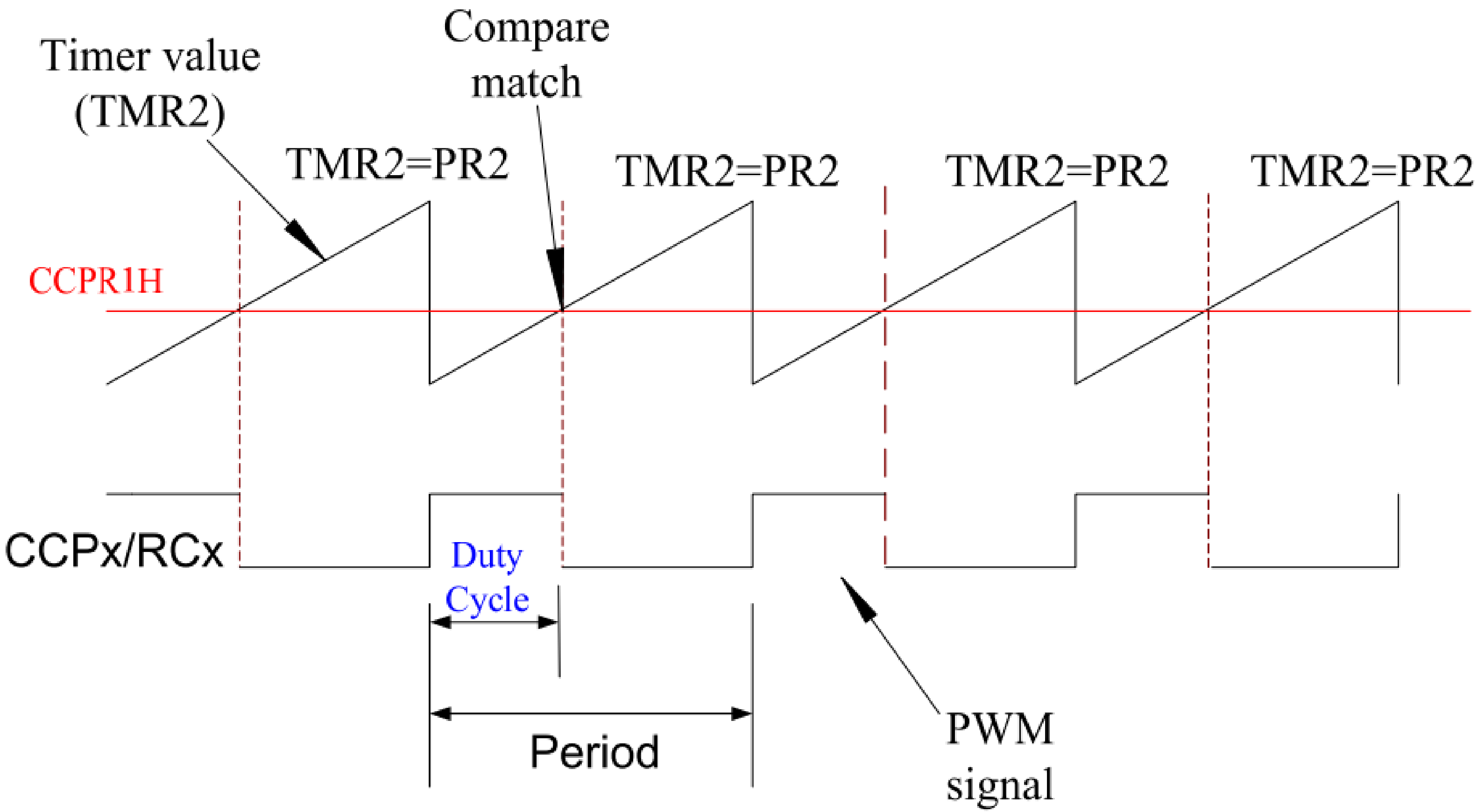


Small time ratio

Department of Electrical Engineering

Pulse-Width Modulation (PWM)

- PWM generated by MCU (Example)



Sinusoidal Pulse Width Modulation (SPWM)

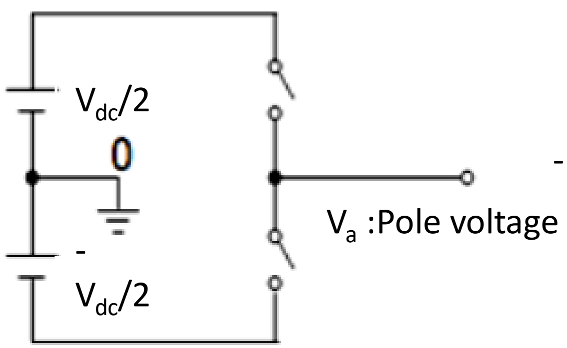
Sinusoidal Pulse Width Modulation (SPWM)

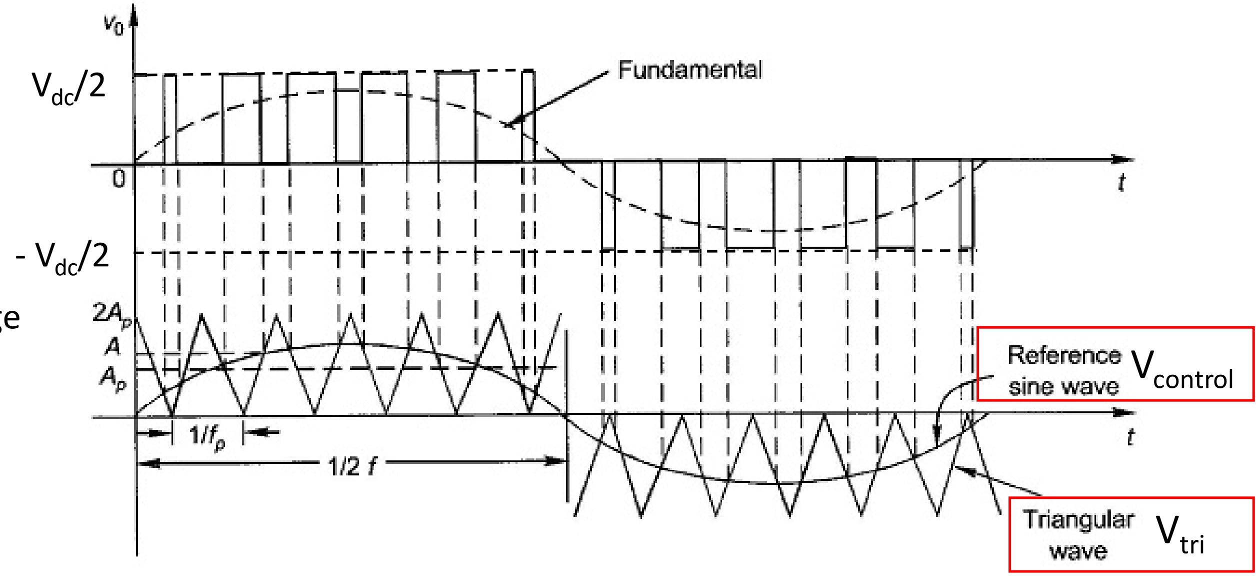
- With advances in solid-state power electronic devices and microprocessors, various inverter control techniques employing pulse width modulation (PWM) techniques are becoming increasingly popular in AC motor drive application, especially with PMSM. These PWM-based drives are used to control both the frequency and the magnitude of the voltage applied to motors.
- In this chapter, Sinusoidal PWM (SPWM) and Space vector PWM (SVPWM) will be introduced and compared about definition, principle, harmonic and efficiency…

Sinusoidal Pulse Width Modulation (SPWM)

Sinusoidal PWM is a type of "carrier-based" pulse width modulation, carrier based PWM uses pre-defined modulation signals to determine output voltages.

SPWM schemes generate the switching position patterns by comparing *reference sine wave* signal with *a carrier triangular wave* signal.





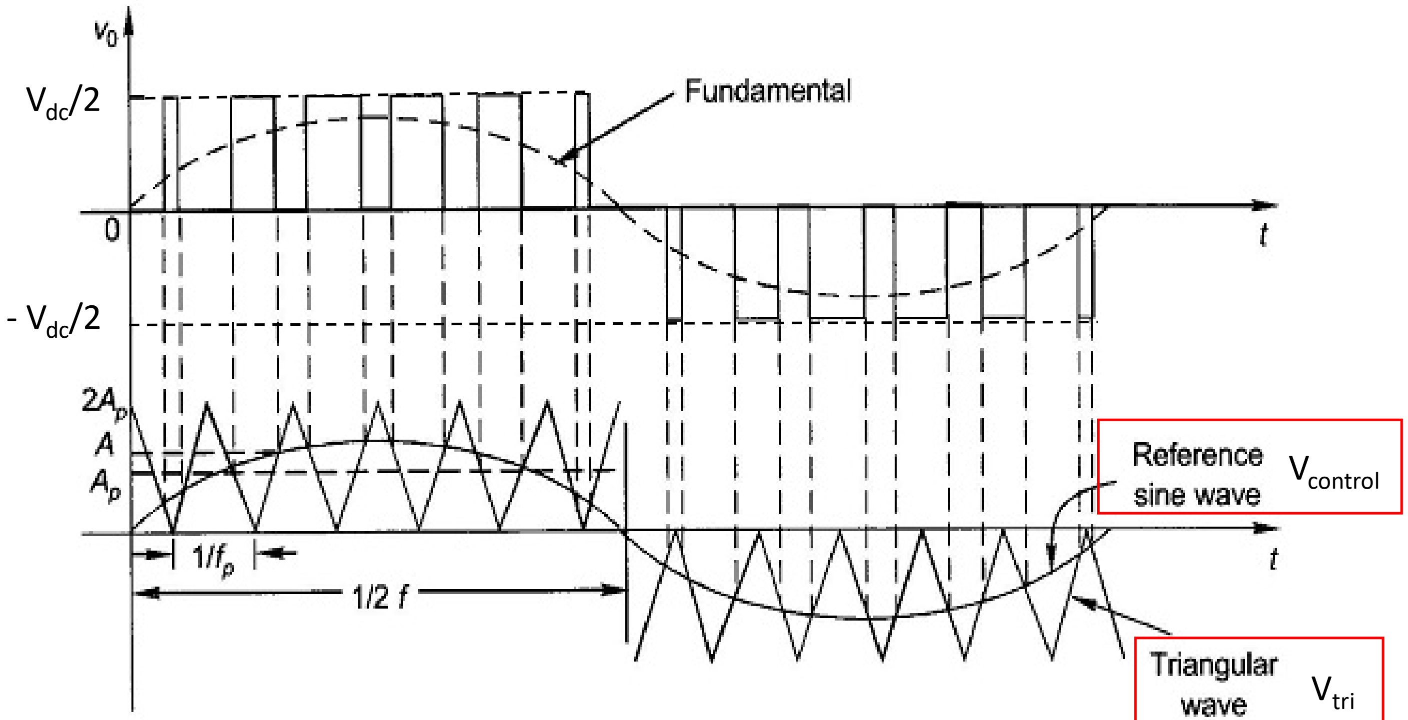
****Considering the circuit model of a single-phase inverter with a centre-taped grounded DC bus illustrate principle of pulse width modulation.*

Principle of SPWM

The inverter output voltage is determined in the following

When $V_{control} > V_{tri}$, $V_{ao} = V_{dc}/2$

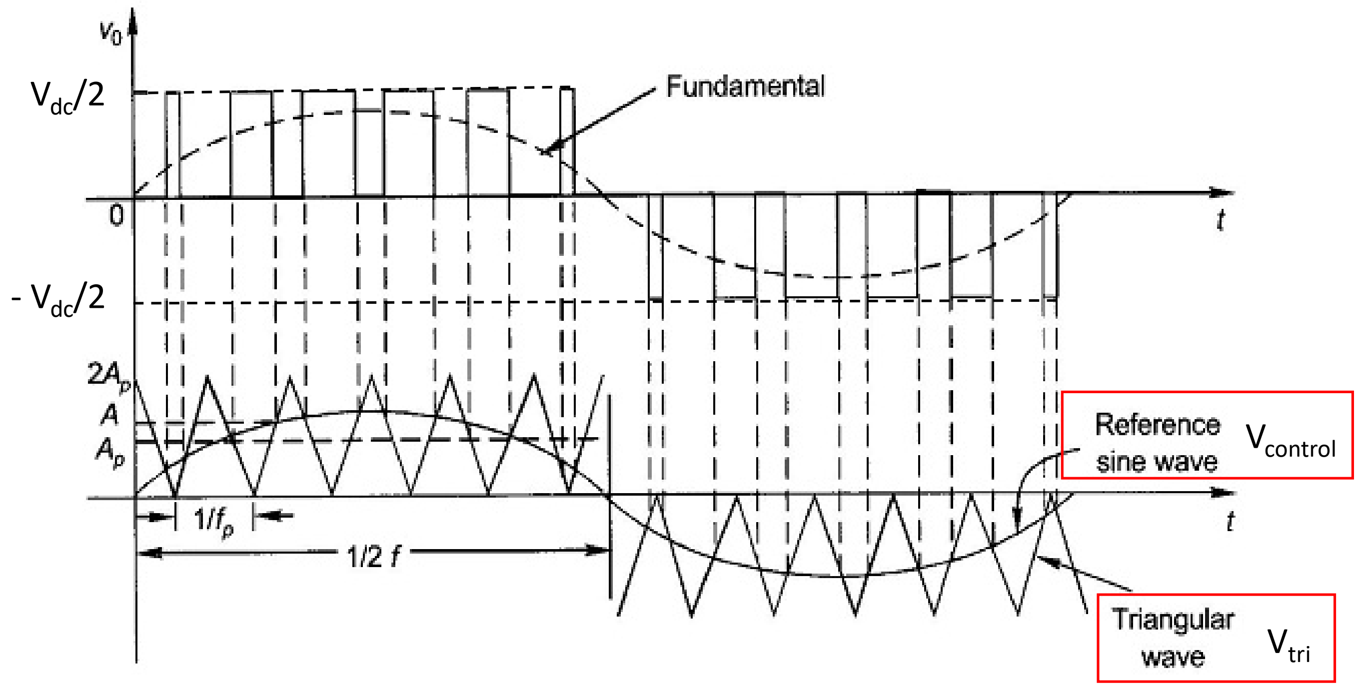
When $V_{control} < V_{tri}$, $V_{ao} = -V_{dc}/2$



Principle of SPWM

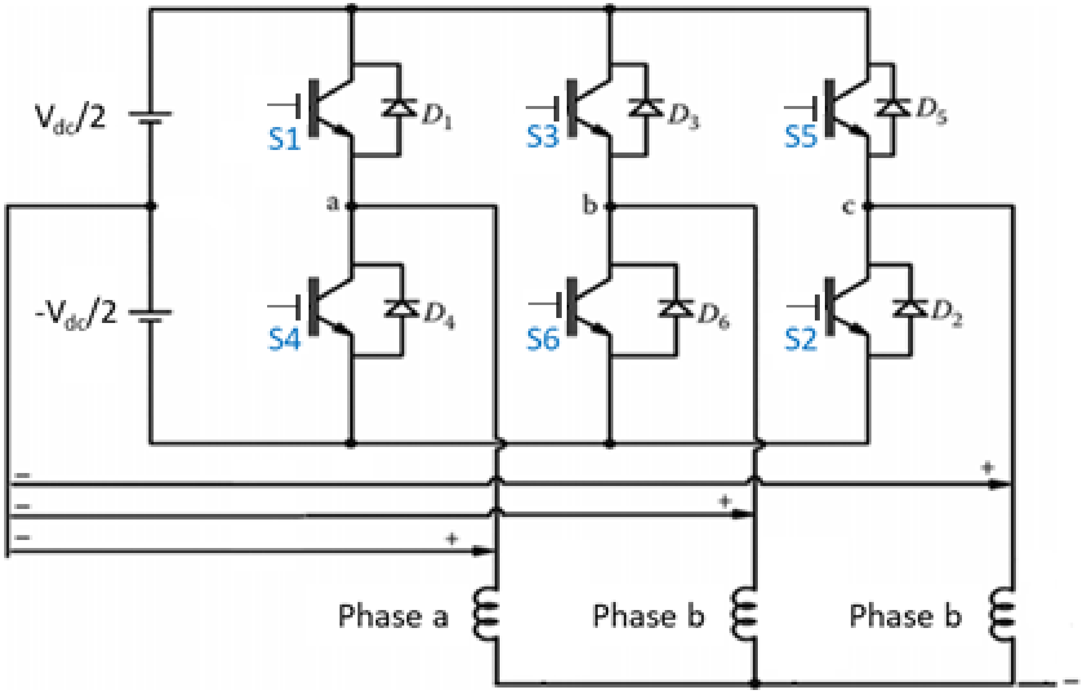
The inverter output voltage has the following features:

- PWM frequency is the same as the frequency of V_{tri}
- Amplitude is controlled by the peak value of $V_{control}$
- Fundamental frequency is controlled by the frequency of $V_{control}$



Principle of SPWM

Considering circuit model of three-phase PWM inverter shows waveforms of carrier wave signal (V_{tri}) and control signal ($V_{control}$), inverter output line to neutral voltages are V_{a0}, V_{b0}, V_{b0}, inverter output line to line voltages are V_{ab}, V_{bc}, V_{ca} respectively.



Principle of SPWM

Inverter output voltage

For Phase a:

If $V_{control} > V_{triangle}$ then $V_{ao} = V_{dc/2.}$

If $V_{control} < V_{triangle}$ then $V_{ao} = - V_{dc/2}$.

For Phase b:

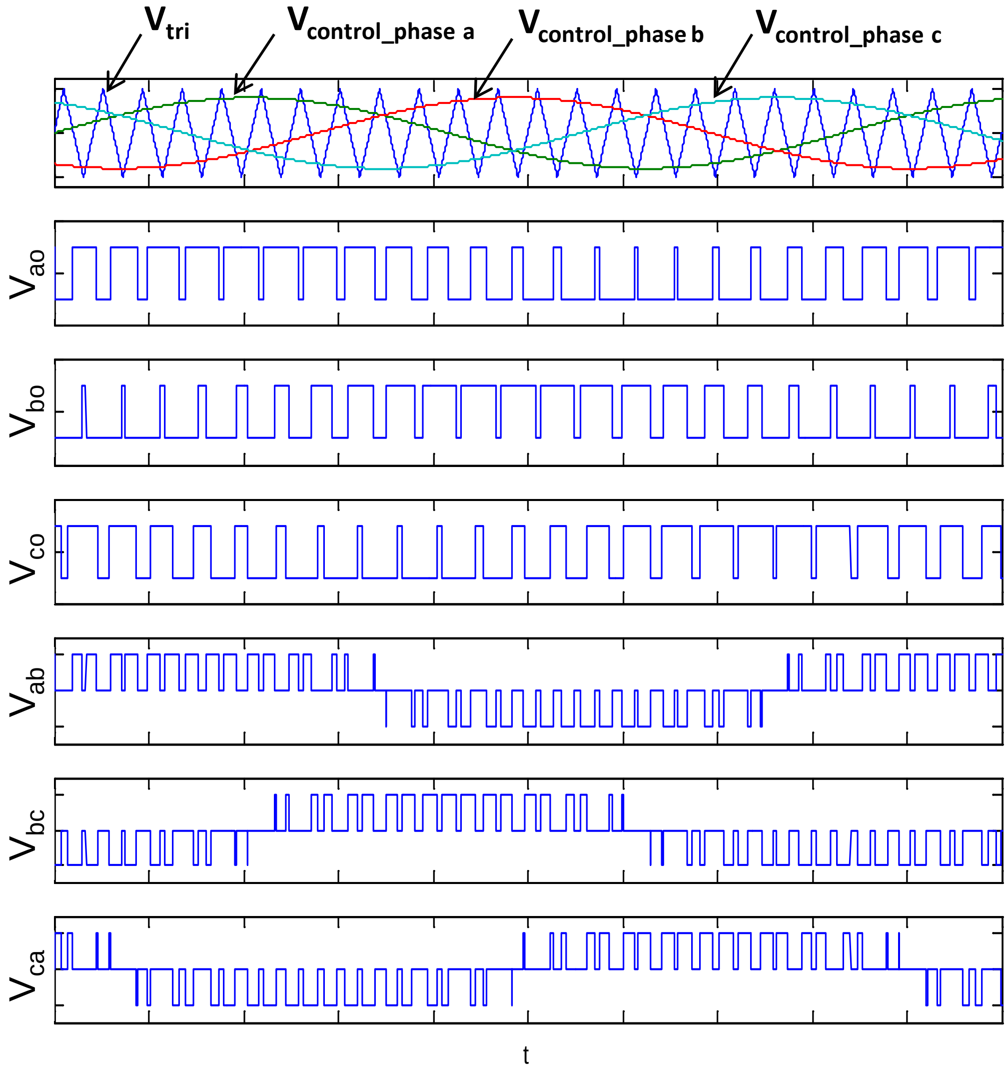
If $V_{control} > V_{triangle}$ then $V_{bo} = V_{d/c2}$.

If $V_{control} < V_{triangle}$ then $V_{bo} = - V_{dc/2}$.

For Phase c:

If $V_{control} > V_{triangle}$ then $V_{co} = V_{dc/2}$.

If $V_{control} < V_{triangle}$ then $V_{co} = - V_{dc/2}$.



Space Vector Pulse Width Modulation (SVPWM)

Space Vector Pulse Width Modulation (SVPWM)

The working principle of SVPWM is to synthesize the stator current to be generated by using the basic voltage vector of the three-phase PWM Inverter. This combined current generates a rotating stator flux vector on the stator coil and interacts with the rotor flux to generate torque, which causes the motor to rotate.

By controlling the voltage vector, the motor air gap rotating flux vector trajectory approaches an ideal circle with minimal flux chopping. Since the torque ripple is the lowest, the speed ripple is also minimized in the case of open circuit control.

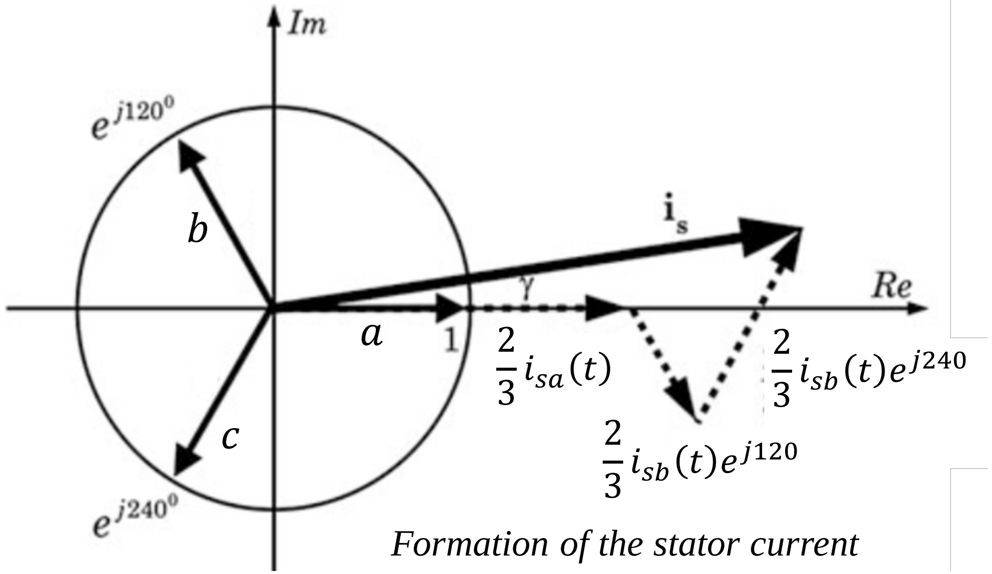
Formation of the Space Vectors

The three sinusoidal phase currents i_a, i_b and i_c of a neutral point isolated 3-phase AC machine fulfill the following relation:

$$i_a(t) + i_b(t) + i_c(t) = 0$$

These currents can be combined to a vector is(t) circulating with the stator frequency f_s

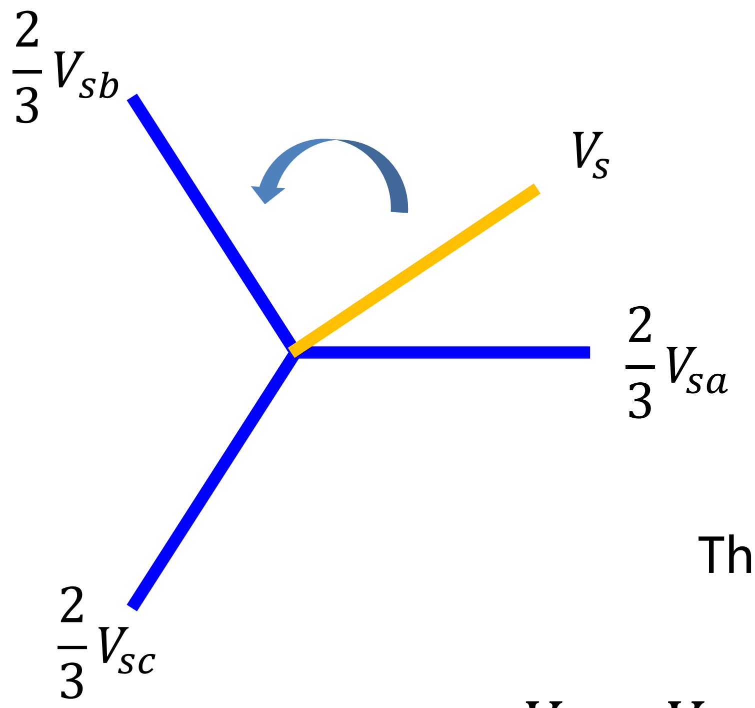
$$i_s = \frac{2}{3}\left[i_{sa}(t) + i_{sb}(t)e^{j\gamma} + i_{sc}(t)e^{j2\gamma}\right], \qquad with\, \gamma = \frac{2\pi}{3}$$



Formation of the stator current vector from the phase currents

The three phase currents now represent the projections of the vector i_s on the accompanying winding axes. Using this idea to combine other 3-phase quantities, complex vectors of stator and rotor voltages v_s, v_r and stator and rotor flux linkages ψ_s;ψ_r are obtained. All vectors circulate with the angular speed ω_s.

Formation of the Space Vectors



$$\begin{bmatrix} V_{sa} \\ V_{sb} \\ V_{sc} \end{bmatrix} = V_{dc} \begin{bmatrix} \cos(\omega t) \\ \cos(\omega t - 120) \\ \cos(\omega t + 120) \end{bmatrix}$$

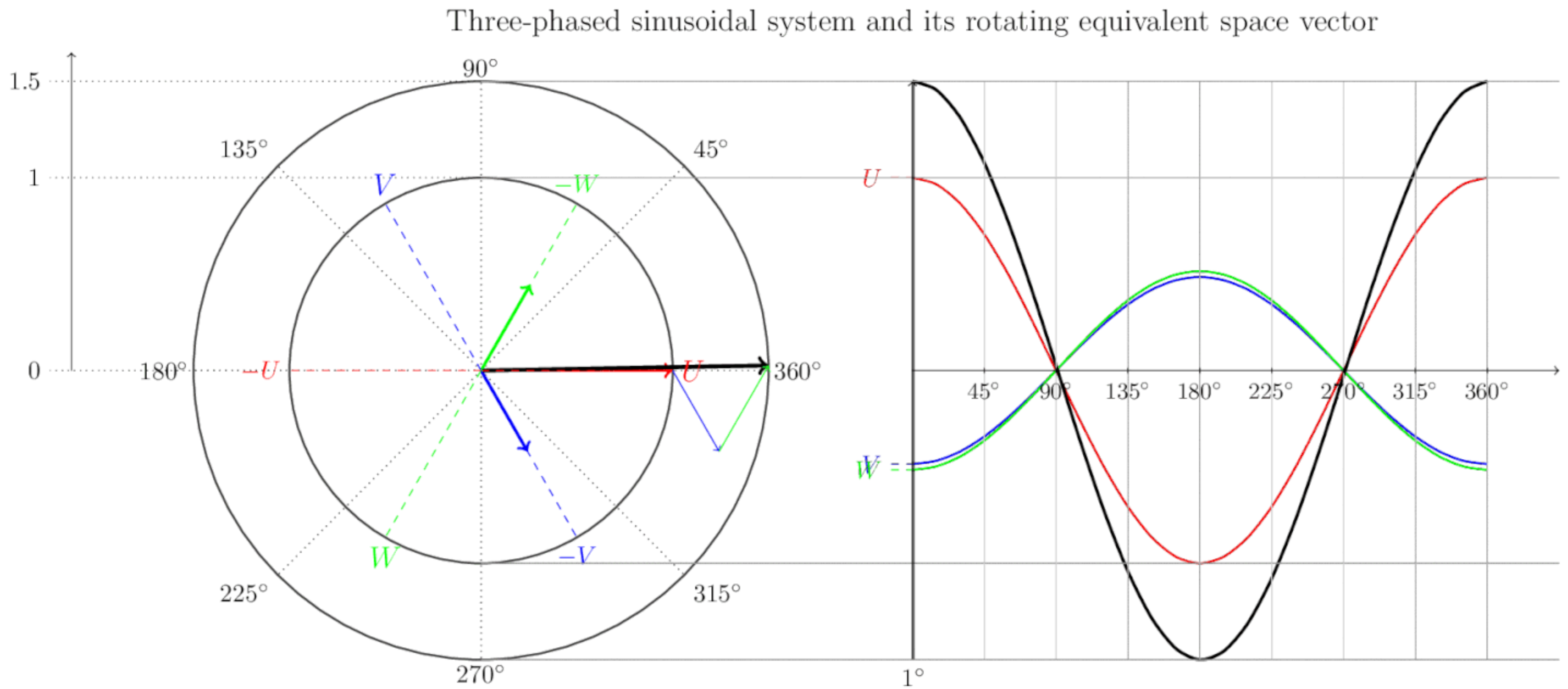
$$V_S = k\left(V_{sa} + V_{sb}e^{-i\frac{2\pi}{3}} + V_{sc}e^{i\frac{2\pi}{3}}\right)$$
$$= k(\tfrac{3}{2}V_{dc}e^{-i\omega t})$$

The amplitude is maintained constant , **k=$\frac{2}{3}$**

$$V_S = V_{dc}e^{-i\omega t} = \left(\frac{2}{3}V_{sa} + \frac{2}{3}V_{sb}e^{-i\frac{2\pi}{3}} + \frac{2}{3}V_{sc}e^{i\frac{2\pi}{3}}\right)$$

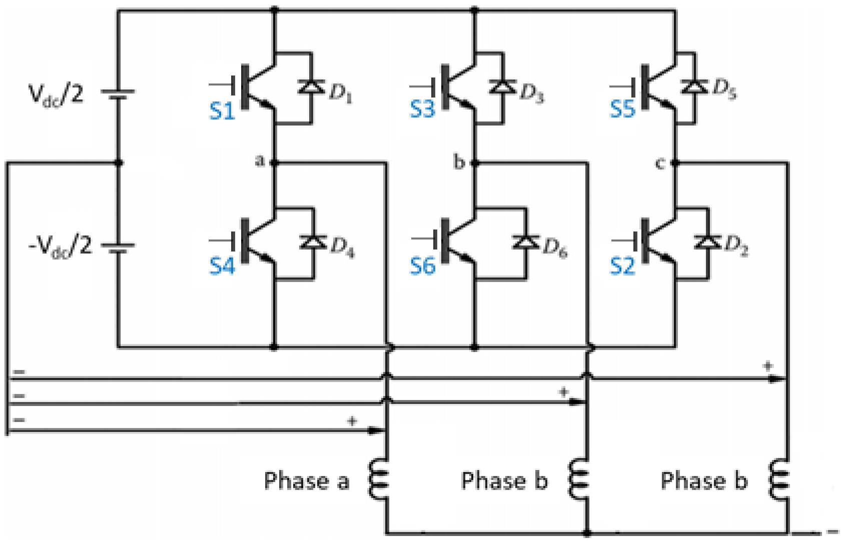
Formation of the Space Vectors

An ordinary three phased system, here shown in both vector form and in sinusoidal form. The **black vector** is the resultant space vector which a vector sum obtained by adding the three vectors. As can be seen, the space vector's magnitude is always constant.



https://www.switchcraft.org/learning/2017/3/15/space-vector-pwm-intro

Principle of SVPWM



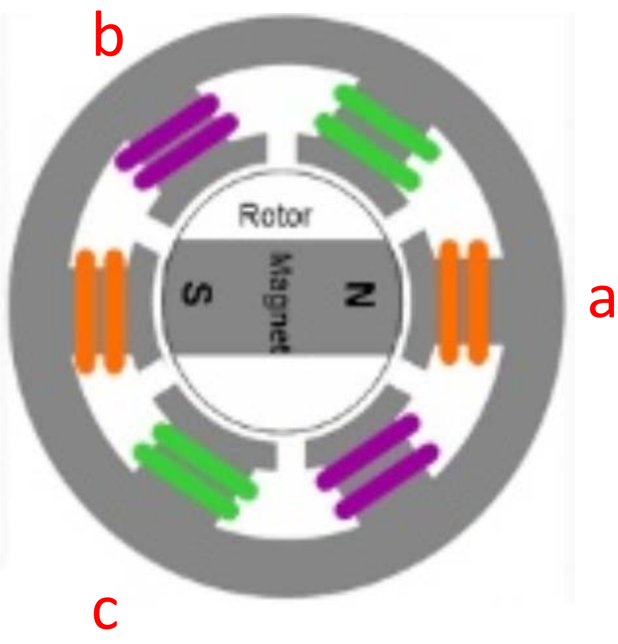
- When an upper switch is turned on (i.e., a, b or c is "1"), the corresponding lower switch is turned off (i.e., a, b or c is "0")
- Eight voltage vector (V_0, V_1,…V_7) is combinations of on and off patterns for the three upper transistors (S1, S3, S5)

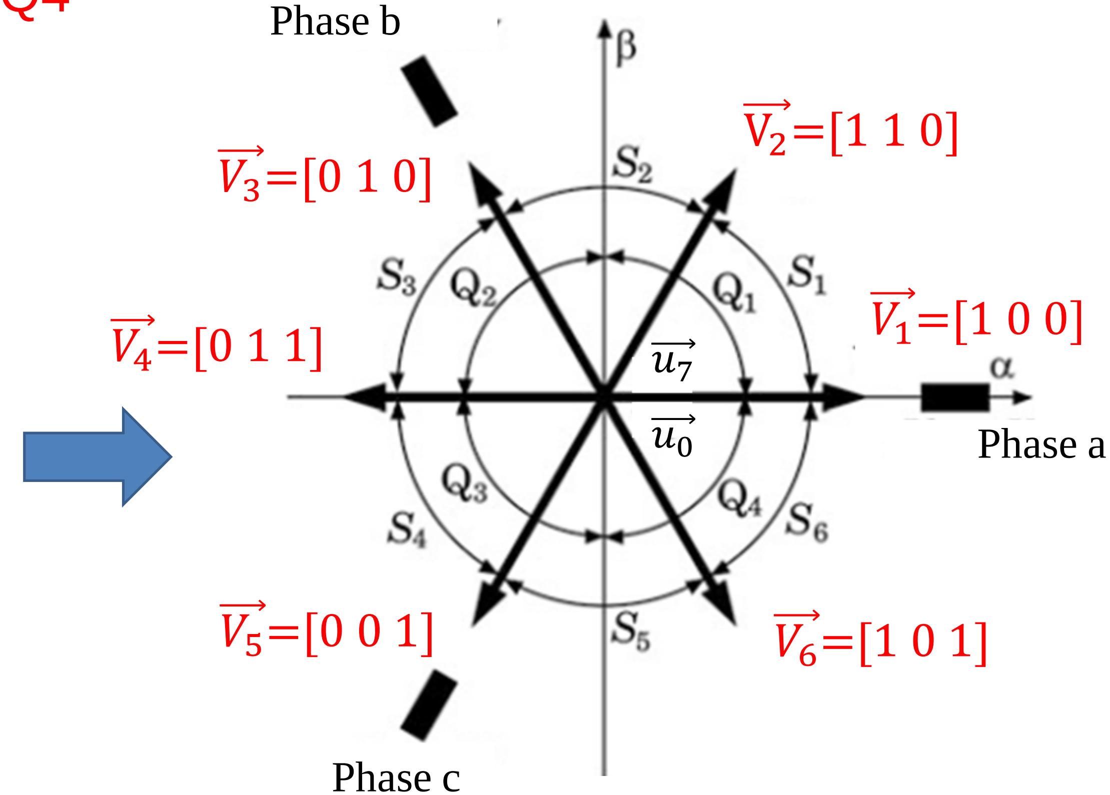
Table 5.1. All ON/ OFF states of upper and lower switch in SVPWM

$\vec{V}_0$=[0 0 0]	$\vec{V}_1$=[1 0 0]	$\vec{V}_2$=[1 1 0]	$\vec{V}_3$=[0 1 0]	$\vec{V}_4$=[0 1 1]	$\vec{V}_5$=[0 0 1]	$\vec{V}_6$=[1 0 1]	$\vec{V}_7$=[1 1 1]
S1:OFF S3:OFF S5:OFF	S1:ON S3:OFF S5:OFF	S1:ON S3:ON S5:OFF	S1:OFF S3:ON S5:OFF	S1:OFF S3:ON S5:ON	S1:OFF S3:OFF S5:ON	S1:ON S3:OFF S5:ON	S1:ON S3:ON S5:ON
S4:ON S6:ON S2:ON	S4:OFF S6:ON S2:ON	S4:OFF S6:OFF S2:ON	S4:ON S6:OFF S2:ON	S4:ON S6:OFF S2:OFF	S4:ON S6:ON S2:OFF	S4:OFF S6:ON S2:OFF	S4:OFF S6:OFF S2:OFF

Principle of SVPWM

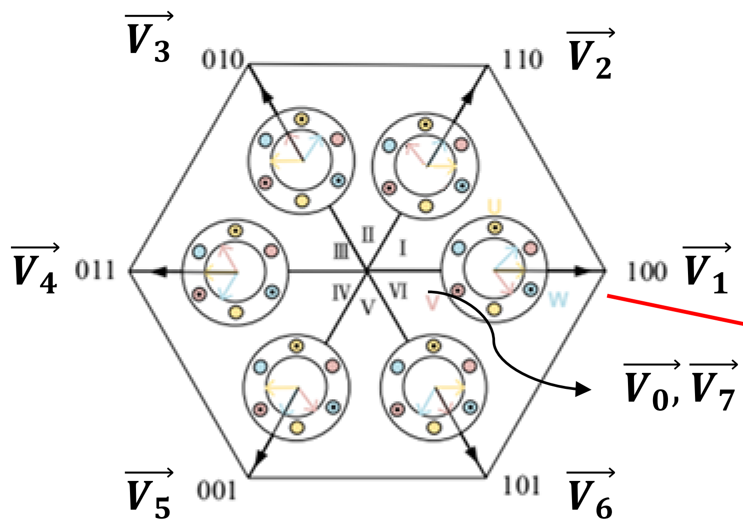
The spacial positions of the standard voltage vectors $\vec{V}_0 \ldots \vec{V}_7$ in stator-fixed αβ coordinates in relation to the three windings a, c and c. The vectors divide the vector space into six sectors S1… S6 and respectively into four quadrants Q1… Q4

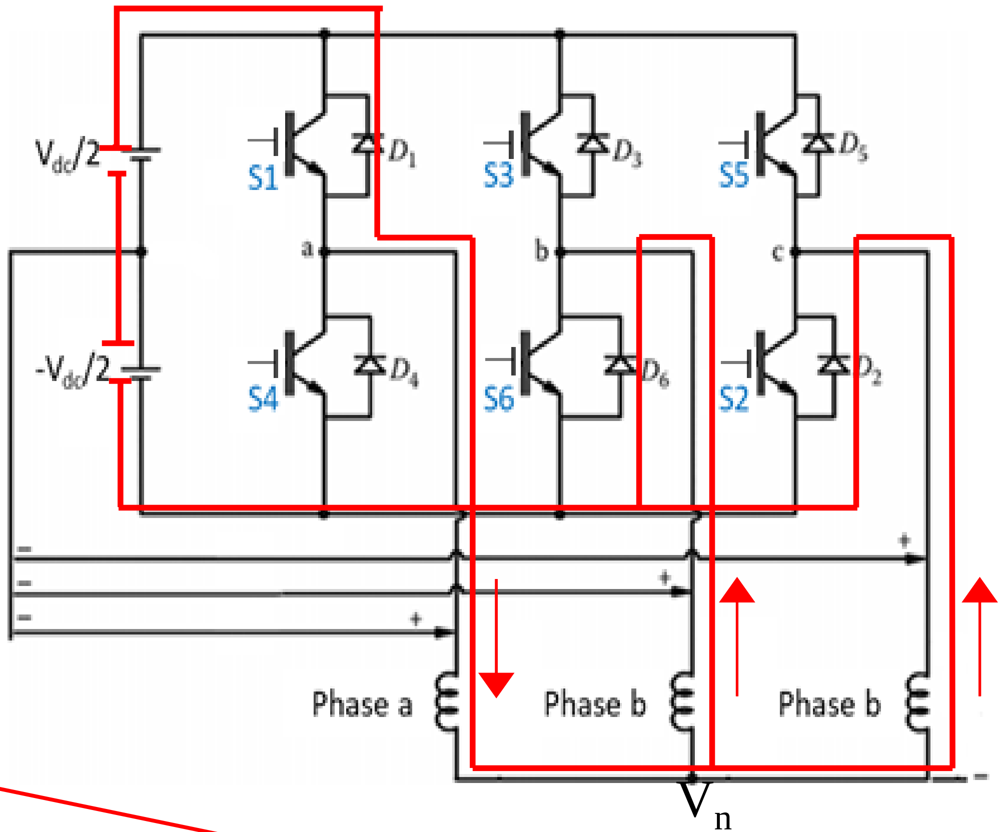




Principle of SVPWM

- V_{xn} (*with x= a, b, c*) represents the phase voltage of the x phase, V_{ab}, V_{bc}, V_{ca} represent line voltage
- n is the neutral point of the motor





$V_{an} = 2V_{dc}/3$

$V_{bn}, V_{cn} = -V_{dc}/3$

Table 5.2. The Standard Voltage Vectors and Logic State

No.	Switching state sequence	V_a	V_b	V_c	V_{ab}	V_{bc}	$\vec{V}$
$\vec{V_0}$	S2, S4, S6	0	0	0	0	0	0
$\vec{V_1}$	S6, S1, S2	$2V_{dc}/3$	$V_{dc}/3$	$-V_{dc}/3$	V_{dc}	0	$2V_{dc}/3\angle 0$
$\vec{V_2}$	S1, S2, S3	$V_{dc}/3$	$V_{dc}/3$	$-2V_{dc}/3$	0	V_{dc}	$2V_{dc}/3\angle(\frac{\pi}{3})$
$\vec{V_3}$	S2, S3, S4	$-V_{dc}/3$	$2V_{dc}/3$	$-V_{dc}/3$	$-V_{dc}$	V_{dc}	$2V_{dc}/3\angle(\frac{\pi}{3})$
$\vec{V_4}$	S3, S4, S5	$-2V_{dc}/3$	$V_{dc}/3$	$V_{dc}/3$	$-V_{dc}$	0	$2V_{dc}/3\angle(-\pi)$
$\vec{V_5}$	S4, S5, S6	$-V_{dc}/3$	$-V_{dc}/3$	$2V_{dc}/3$	0	$-V_{dc}$	$2V_{dc}/3\angle(-\frac{2\pi}{3})$
$\vec{V_6}$	S5, S6, S1	$V_{dc}/3$	$-2V_{dc}/3$	$V_{dc}/3$	V_{dc}	$-V_{dc}$	$2V_{dc}/3\angle(-\frac{\pi}{3})$
$\vec{V_7}$	S1, S3, S5	0	0	0	0	0	0

Principle of SVPWM

- Let us assume that the vector to be realized, V_s is located in the sector S1, the area between the standard vectors V_1 and V_2. V_s can be obtained from the vectorial addition of the two boundary vectors V_r and V_l in the directions of V_1 and V_2, respectively.
- Supposed the complete pulse period T_p^* is available for the realization of a vector with the maximum modulus (amplitude), which corresponds to the value $2V_{dc}/3$ of a standard vector, the following relation is valid:

$$|\boldsymbol{V}_s|_{\max} = |\boldsymbol{V}_1| = \cdots = |\boldsymbol{V}_6| = 2V_{dc}/3$$

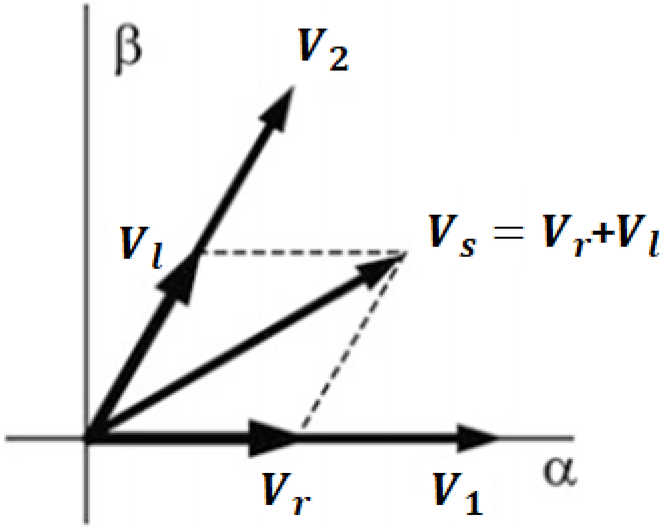
- V_r and V_l are realized by the logical states of the vectors V_1 and V_2 within the time span:

$$T_r = T_p^* \frac{|V_r|}{|V_s|_{max}} \quad T_l = T_p^* \frac{|V_l|}{|V_s|_{max}}$$

T_r , T_l are switching times must be calculated.

To be able to determine T_r and T_l, the amplitudes of V_r and V_l must be known.

*(***Subscript r, l: boundary vector on the right, left)*



Realization of an arbitrary voltage vector from two boundary vectors

Principle of SVPWM

- It is prerequisite that the stator voltage vector V_s must be provided by the current controller with respect to modulus and phase.
- In the rest of the pulse period $T_p^* - (T_r + T_l)$ one of the two zero vectors V_0 or V_7 will be issued to finally fulfil the following equation.

$$\boldsymbol{V_s = V_r + V_l + V_0\ (or\ V_7) = \frac{T_r}{T_p^*} V_1 + \frac{T_l}{T_p^*} V_2 + \frac{T_p^* - (T_r + T_l)}{T_p^*} V_0\ \ (or\ V_7)}$$

- In which sequence the now three vectors—two boundary vectors and one zero vector—must be issued. Table shows the necessary switching states in the sector S1.

Table 5.3. The necessary switching states in the sector S1

Voltage	V_0	V_1	V_2	V_7
V_a	0	1	1	1
V_b	0	0	1	1
V_c	0	0	0	1

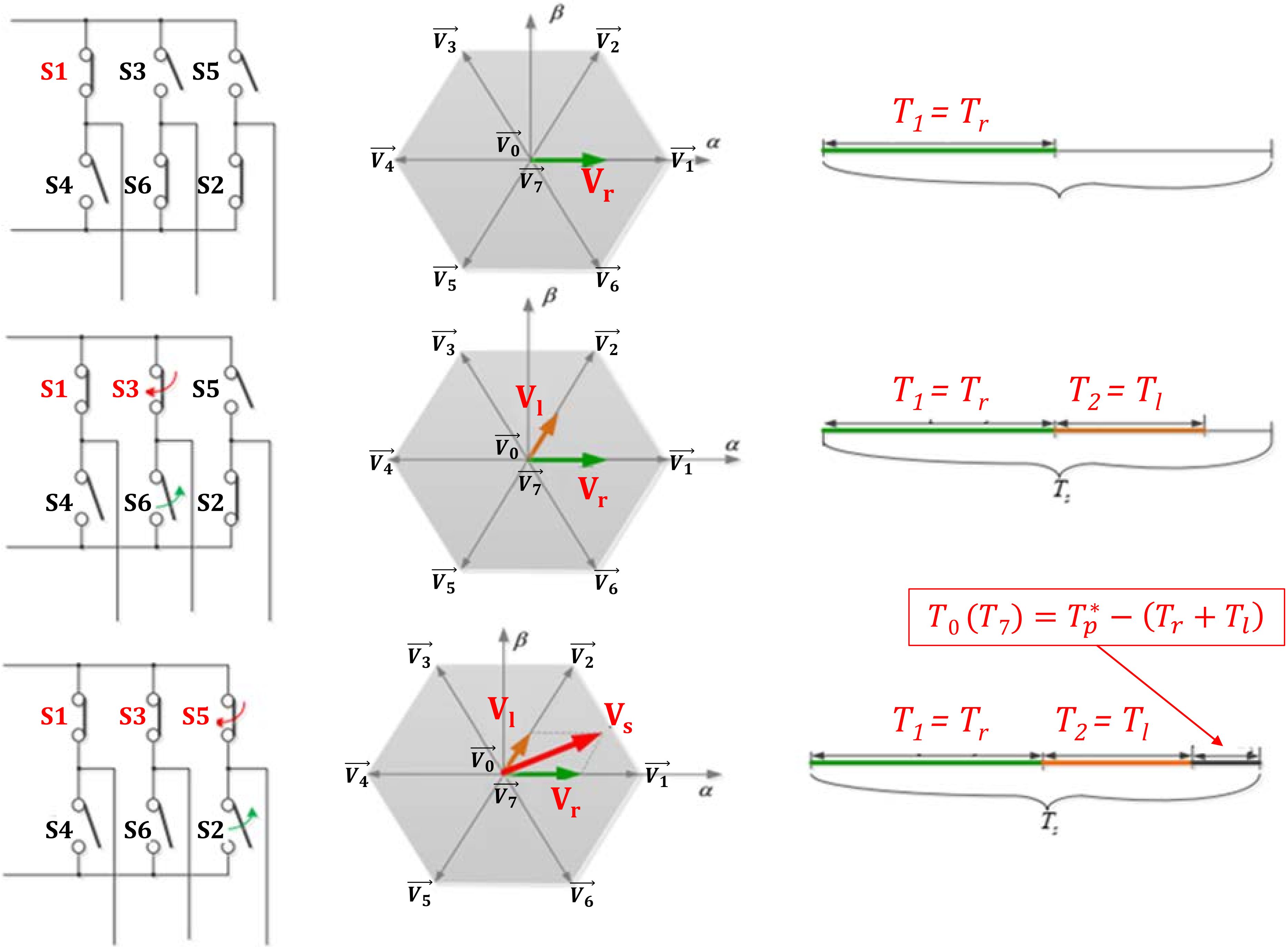
Principle of SVPWM

By observing "Table of switching states in the sector S1", it can be recognized that with respect to transistor switching losses the most favourable sequence is to switch every transistor pair only once within a pulse period.

If the last switching state was V_0, this would be the sequence $V_0 \rightarrow V_1 \rightarrow V_2 \rightarrow V_7$

But if the last switching state was V_7, this would be $V_7 \rightarrow V_2 \rightarrow V_1 \rightarrow V_0$

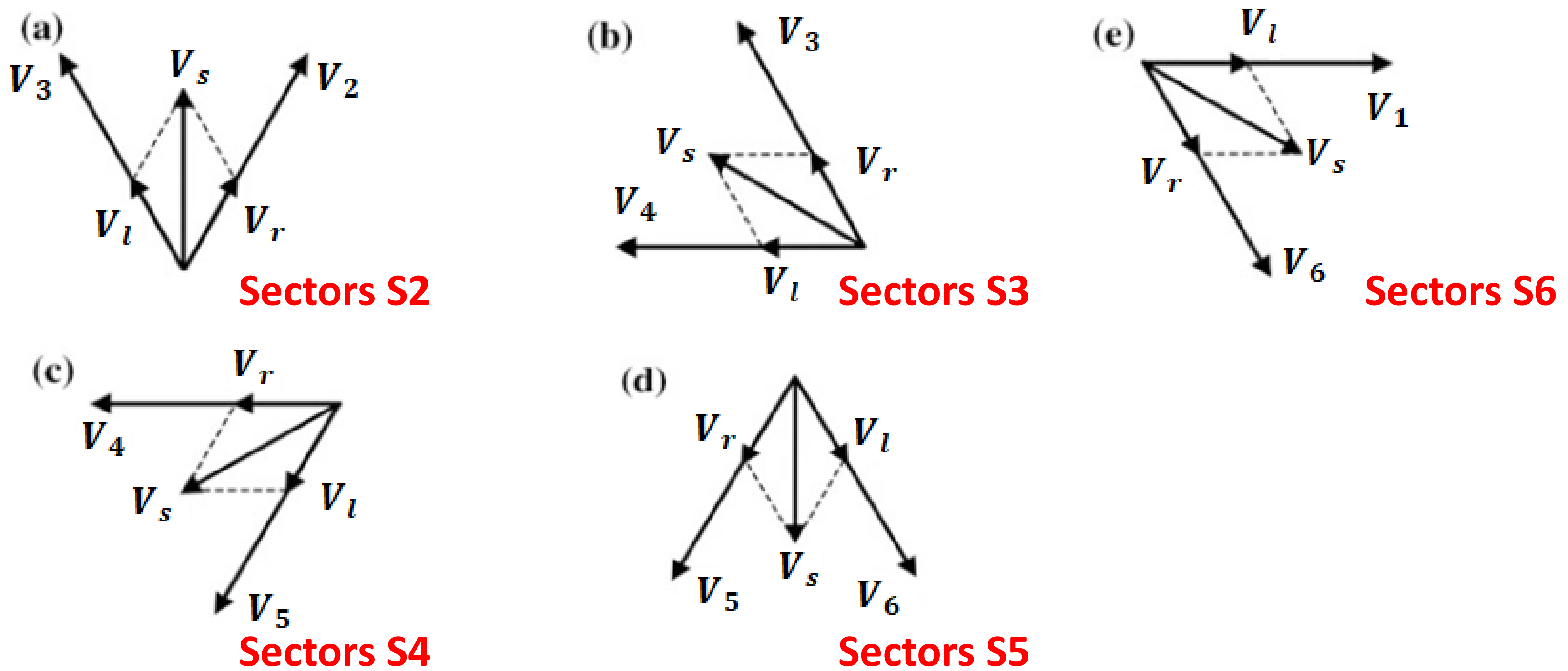
→ ***With this strategy the switching losses of the inverter become minimal.***



S1
S3
S5
S4
S6
S2
β
α
$\vec{V}_0$
$\vec{V}_1$
$\vec{V}_2$
$\vec{V}_3$
$\vec{V}_4$
$\vec{V}_5$
$\vec{V}_6$
$\vec{V}_7$
$\mathbf{V_r}$
$\mathbf{V_l}$
$\mathbf{V_s}$
$T_1 = T_r$
$T_2 = T_l$
$T_0\,(T_7) = T_p^* - (T_r + T_l)$
T_z

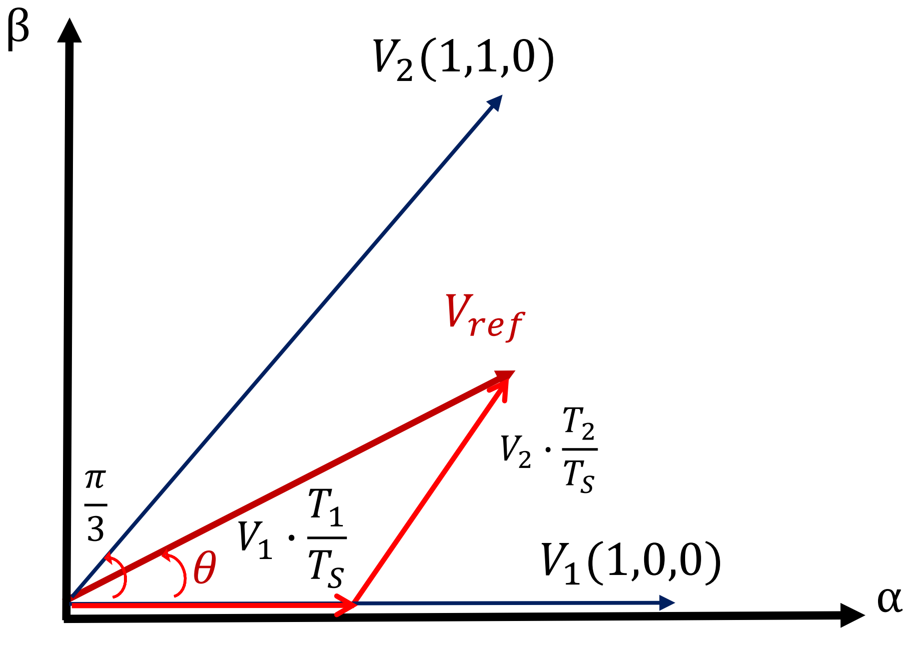
Principle of SVPWM

- The following pictures give a summary of switching pattern samples in the remaining sectors S2 … S6 of the vector space



Pulse pattern of the voltage vectors in the sectors S2… S6

Determined Switching Time



sector I

$$V_1 = \frac{2}{3} V_{dc}$$

$$V_2 = \frac{2}{3} V_{dc} e^{i\frac{\pi}{3}}$$

$$|V_{ref}| = \frac{1}{\sqrt{3}} V_{dc}$$

V_{ref} is a rotating space voltage vector

In sector I , $V_{ref} \cdot T_S = V_1 \cdot T_1 + V_2 \cdot T_2$

T_S: carrier period

$$\begin{bmatrix} |V_{ref}| \cdot cos\theta \\ |V_{ref}| \cdot sin\theta \end{bmatrix} = \begin{bmatrix} V_\alpha \\ V_\beta \end{bmatrix} = \begin{bmatrix} \frac{T_1}{T_S} & cos\frac{\pi}{3} \cdot \frac{T_2}{T_S} \\ 0 & sin\frac{\pi}{3} \cdot \frac{T_2}{T_S} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

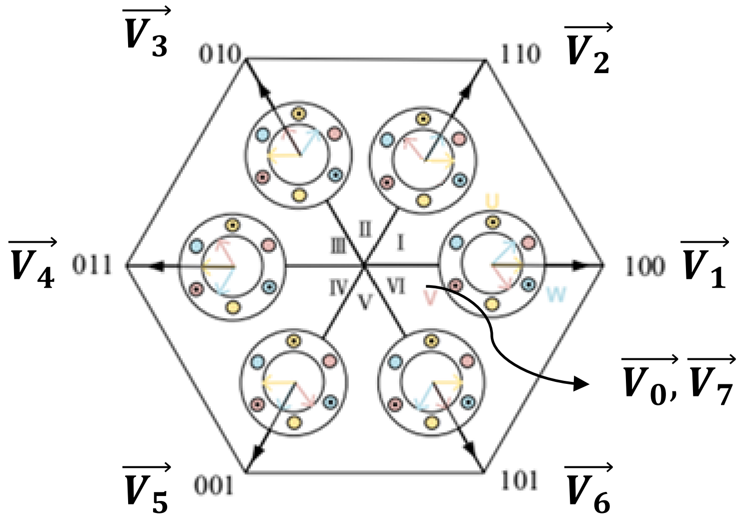
$$\frac{T_1}{T_S} = \sin(\frac{\pi}{3} - \theta)$$

$$\frac{T_2}{T_S} = \sin(\theta)$$

$$\frac{T_1}{T_S} = \frac{1}{V_{dc}}(\frac{3}{2} \cdot V_\alpha - \frac{\sqrt{3}}{2} \cdot V_\beta)$$

$$\frac{T_2}{T_S} = \sqrt{3} \cdot \frac{V_\beta}{V_{dc}}$$

Determined Switching Time



$$X = \frac{1}{V_{dc}} \cdot V_\beta \cdot \sqrt{3}$$

$$Y = \frac{1}{V_{dc}} \cdot (\frac{3}{2} V_\alpha + \frac{\sqrt{3}}{2} V_\beta)$$

$$Z = \frac{1}{V_{dc}} \cdot (\frac{3}{2} V_\alpha - \frac{\sqrt{3}}{2} V_\beta)$$

Table 5.4. Look-up table of sector number

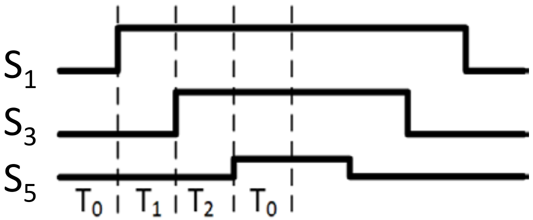
Sector number	I	II	III	IV	V	VI
T_x	Z	Y	X	-Z	-Y	-X
T_y	X	-Z	-Y	-X	Z	Y

T_x:the application time of the first active vector

T_y:the application time of the second active vector

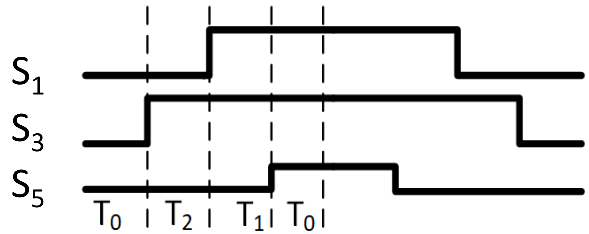
Determined Switching Time

Sector I



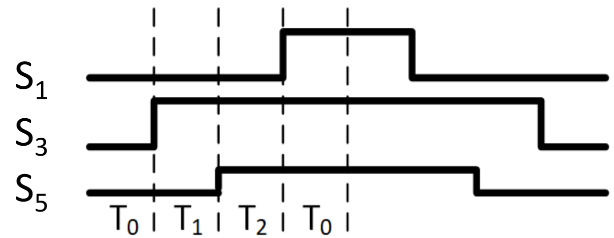
S_1=T1+T2+T0
S_3=T2+T0
S_5=T0

Sector II



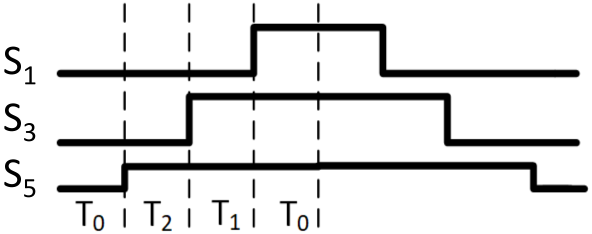
S_1=T1+T0
S_3=T1+T2+T0
S_5=T0

Sector III



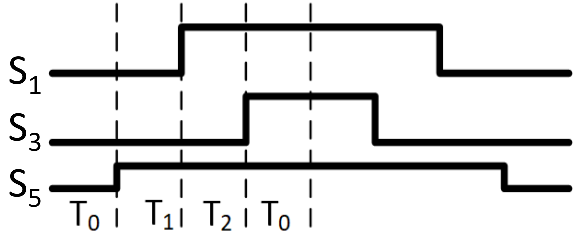
S_1=T0
S_3=T1+T0
S_5=T1+T1+T0

Sector IV



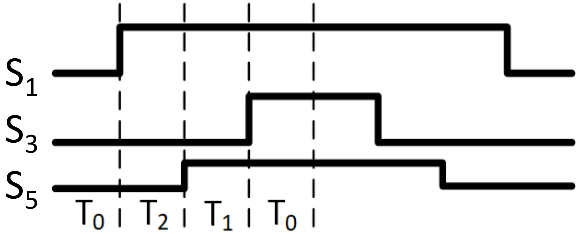
S_1=T0
S_3=T1+T0
S_5=T1+T2+T0

Sector V



S_1=T2+T0
S_3=T0
S_5=T1+T2+T0

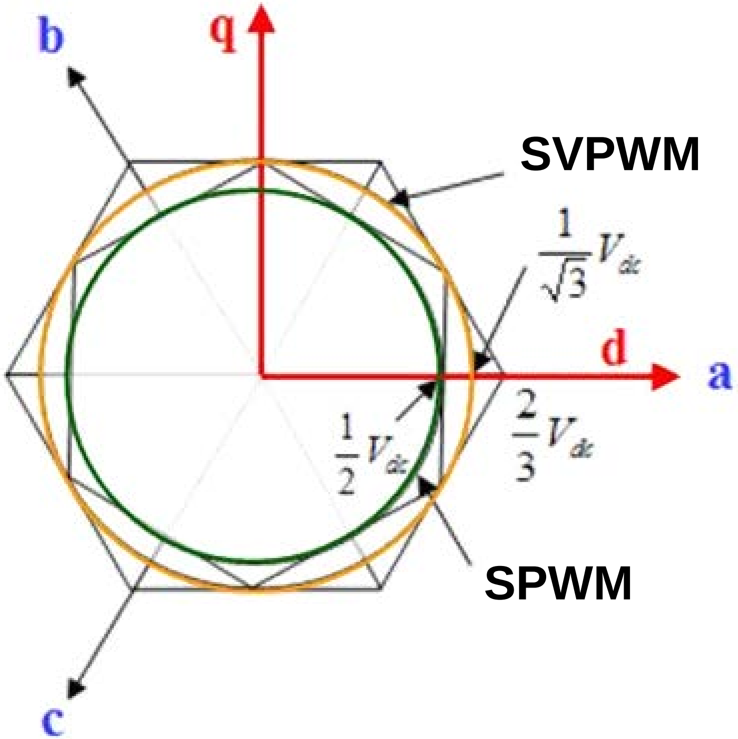
Sector IV



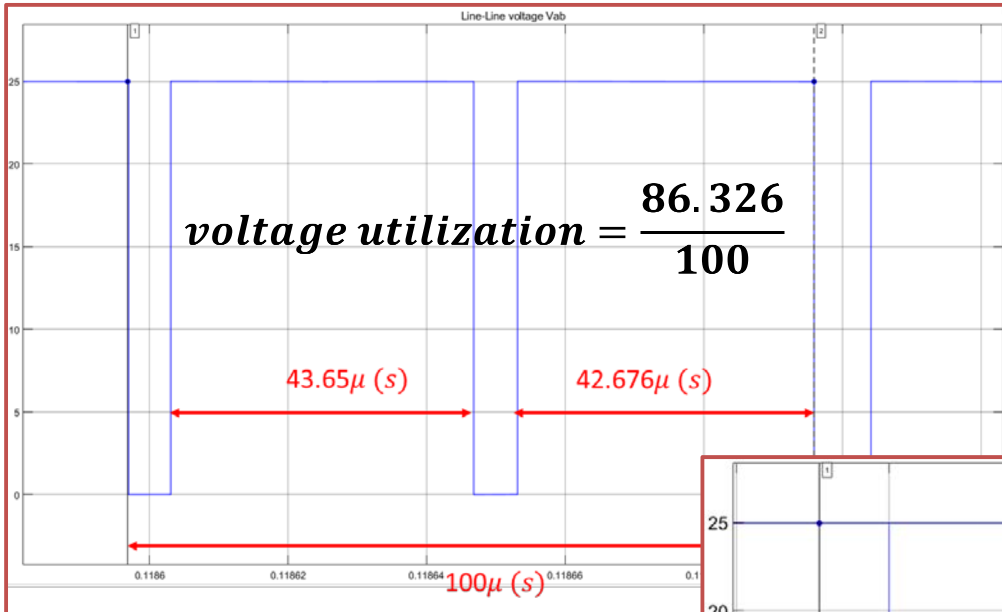
S_1=T1+T2+T0
S_3=T0
S_5=T1+T0

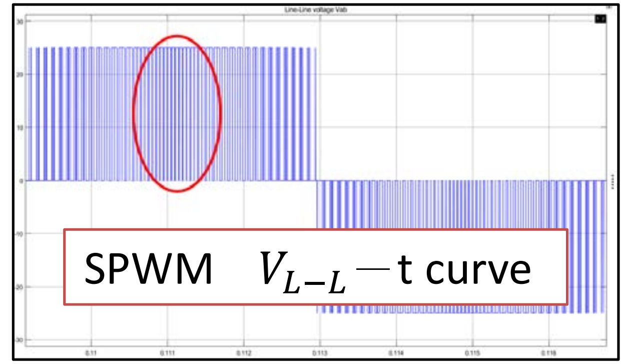
Comparison SPWM and SVPWM

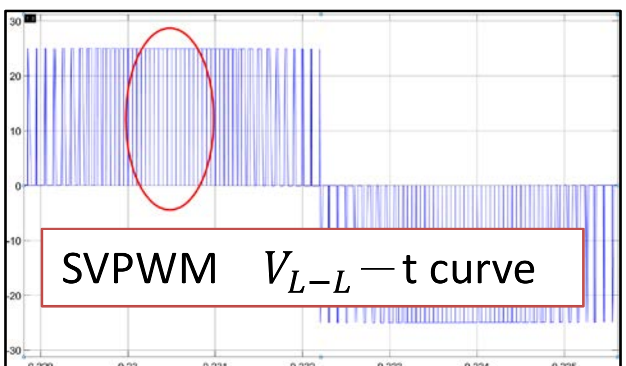
- Sinusoidal PWM (SPWM) : Locus of the reference vector is the inside of a circle with radius of 1/2 Vdc
- Space Vector PWM (SVPWM) : Locus of the reference vector is the inside of a circle with radius of $1/\sqrt{3}$ Vdc

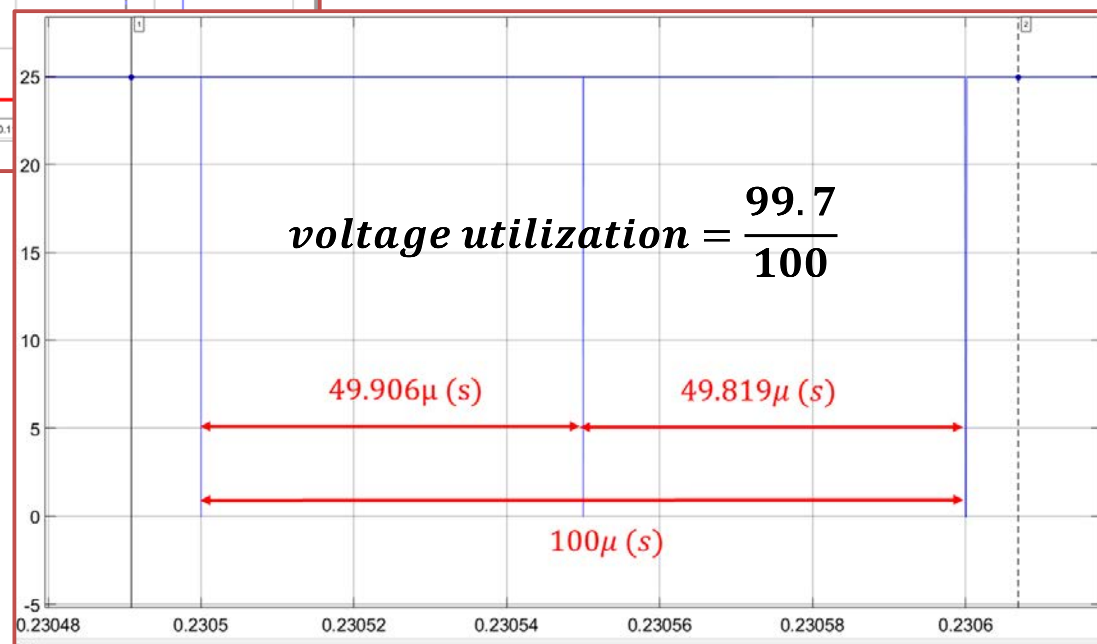


Compare SPWM with SVPWM









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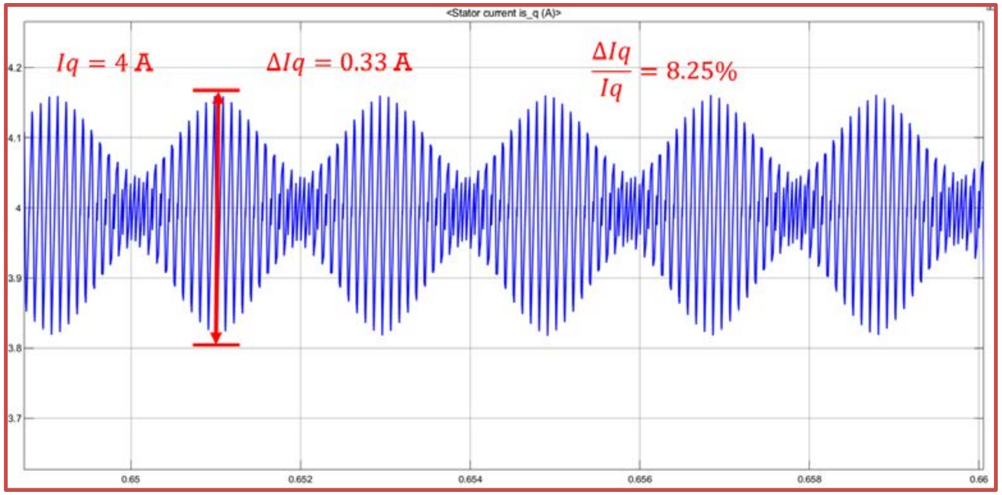
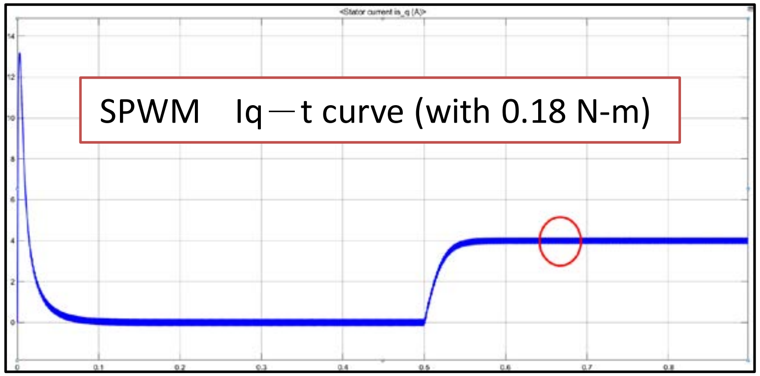
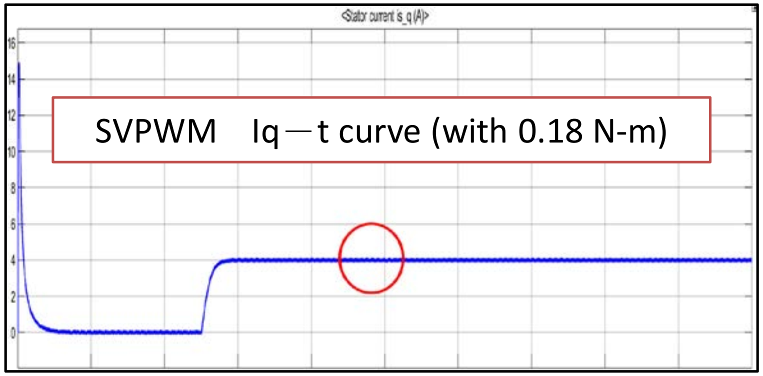
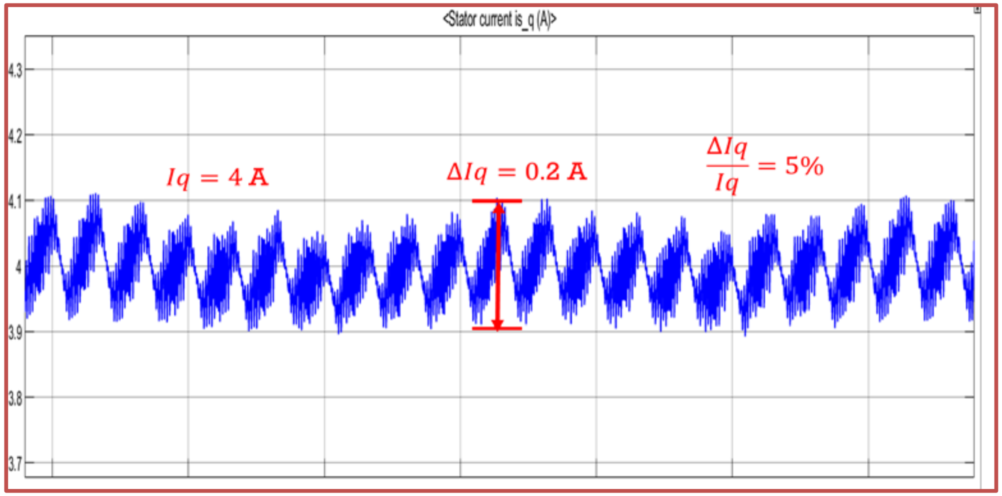
Compare SPWM with SVPWM









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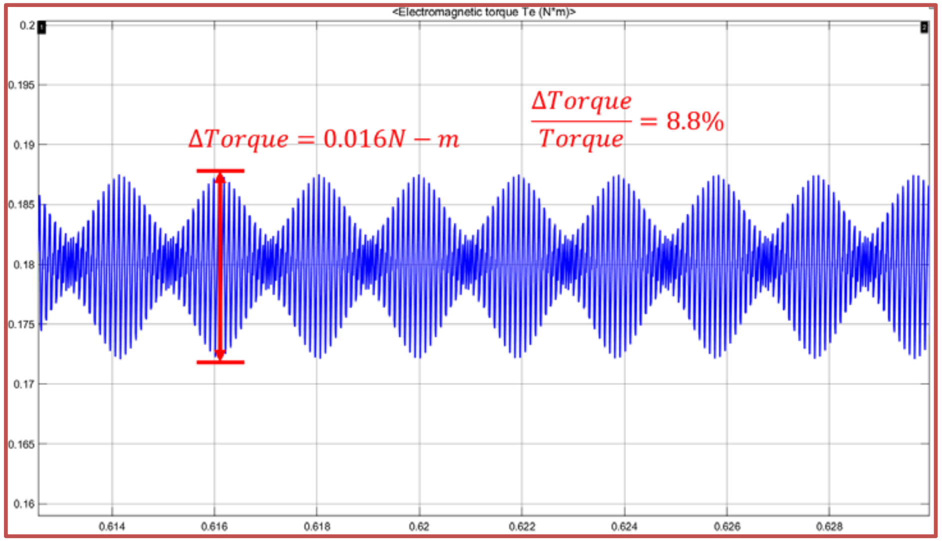
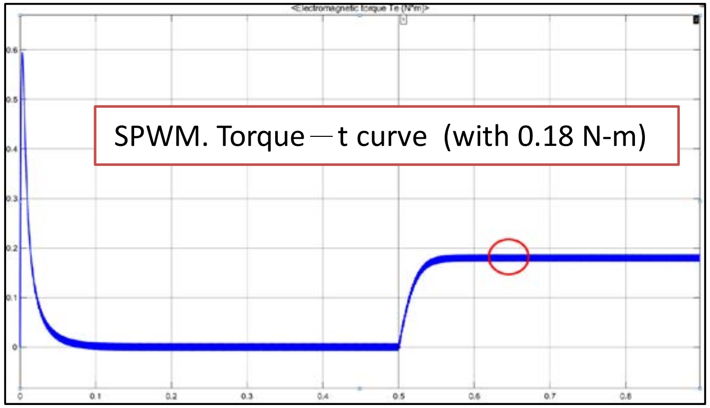
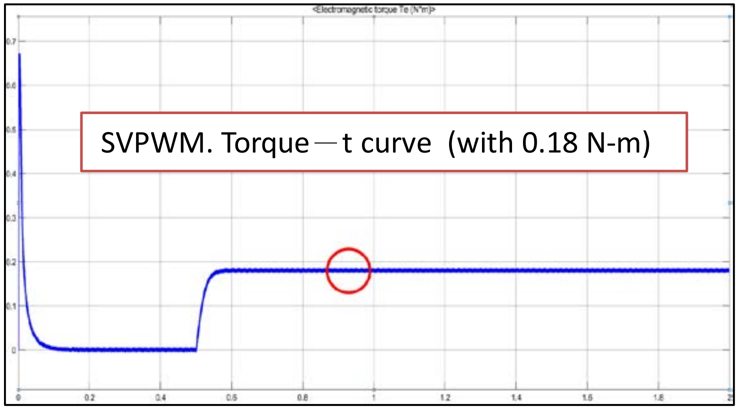
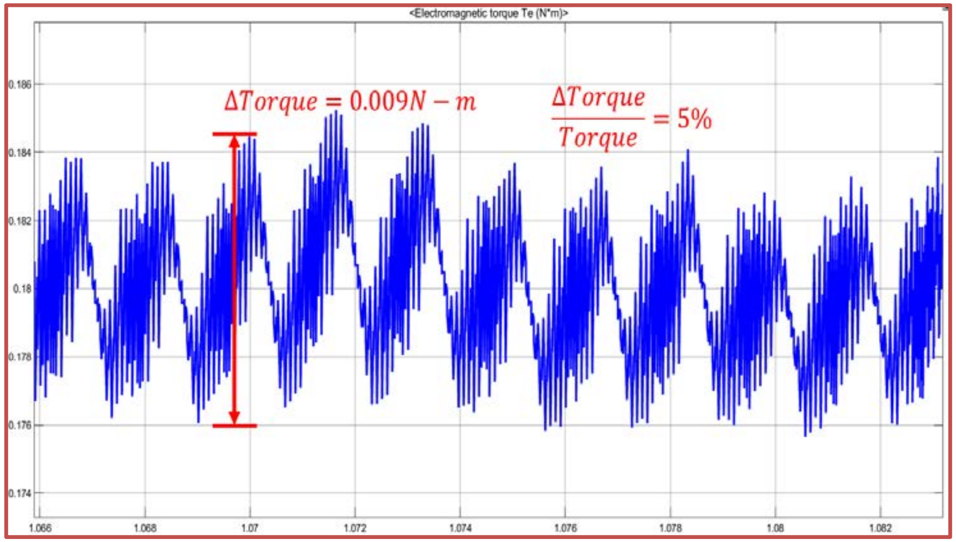
Compare SPWM with SVPWM









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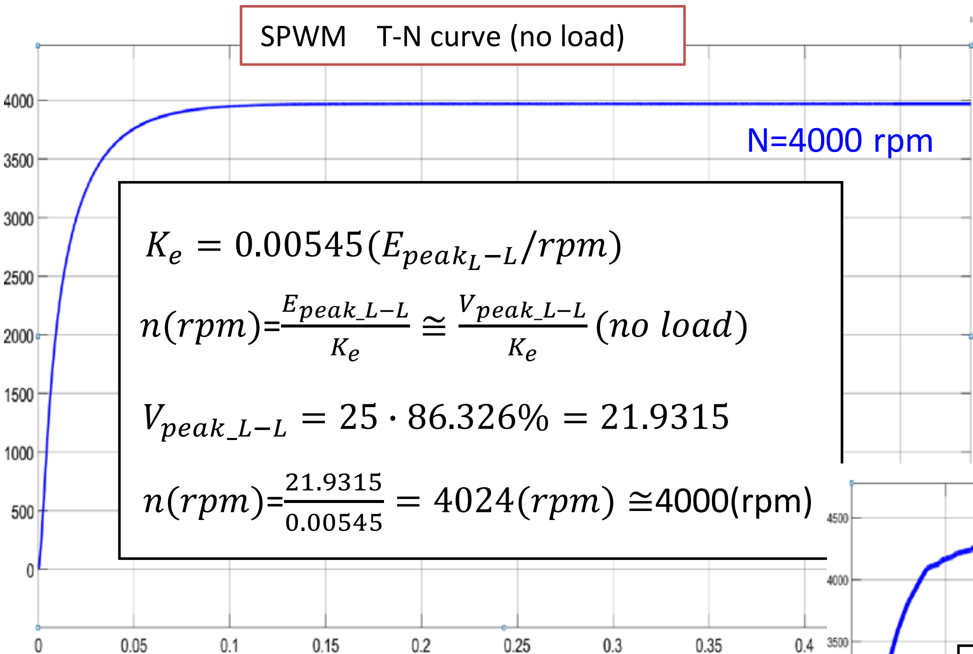
Compare SPWM with SVPWM

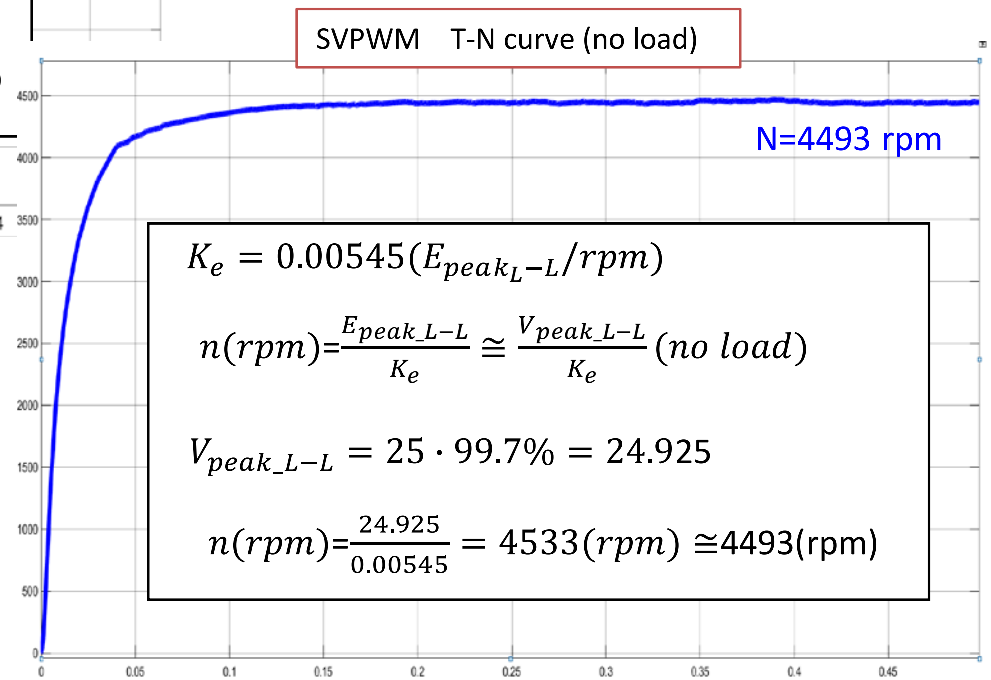


$$K_e = 0.00545(E_{peak_L-L}/rpm)$$

$$n(rpm)=\frac{E_{peak\_L-L}}{K_e} \cong \frac{V_{peak\_L-L}}{K_e}(no\ load)$$

$$V_{peak\_L-L} = 25 \cdot 86.326\% = 21.9315$$

$$n(rpm)=\frac{21.9315}{0.00545} = 4024(rpm) \cong 4000(\text{rpm})$$



$$K_e = 0.00545(E_{peak_L-L}/rpm)$$

$$n(rpm)=\frac{E_{peak\_L-L}}{K_e} \cong \frac{V_{peak\_L-L}}{K_e}(no\ load)$$

$$V_{peak\_L-L} = 25 \cdot 99.7\% = 24.925$$

$$n(rpm)=\frac{24.925}{0.00545} = 4533(rpm) \cong 4493(\text{rpm})$$

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Comparison SPWM and SVPWM

Table 5.5. Result of "Simulation of Compare SPWM with SVPWM",

Item.	SPWM	SVPWM
Voltage utilization	86.326%	99.7%
Speed (no-load)	4000 (rpm)	4493 (rpm)
Torque ripple	8.8%	5%
Current(Iq) ripple	8.25%	5%

- Space Vector PWM generates less harmonic distortion in the output voltage or currents in comparison with sinusoidal PWM.
- Space Vector PWM provides more efficient use of supply voltage in comparison with Sinusoidal PWM
- SPWM can be used for the application where a small compromise can be done with output quality and the application requirements are less complexity. But where better performance with high output quality is desired the choice should be SVPWM.